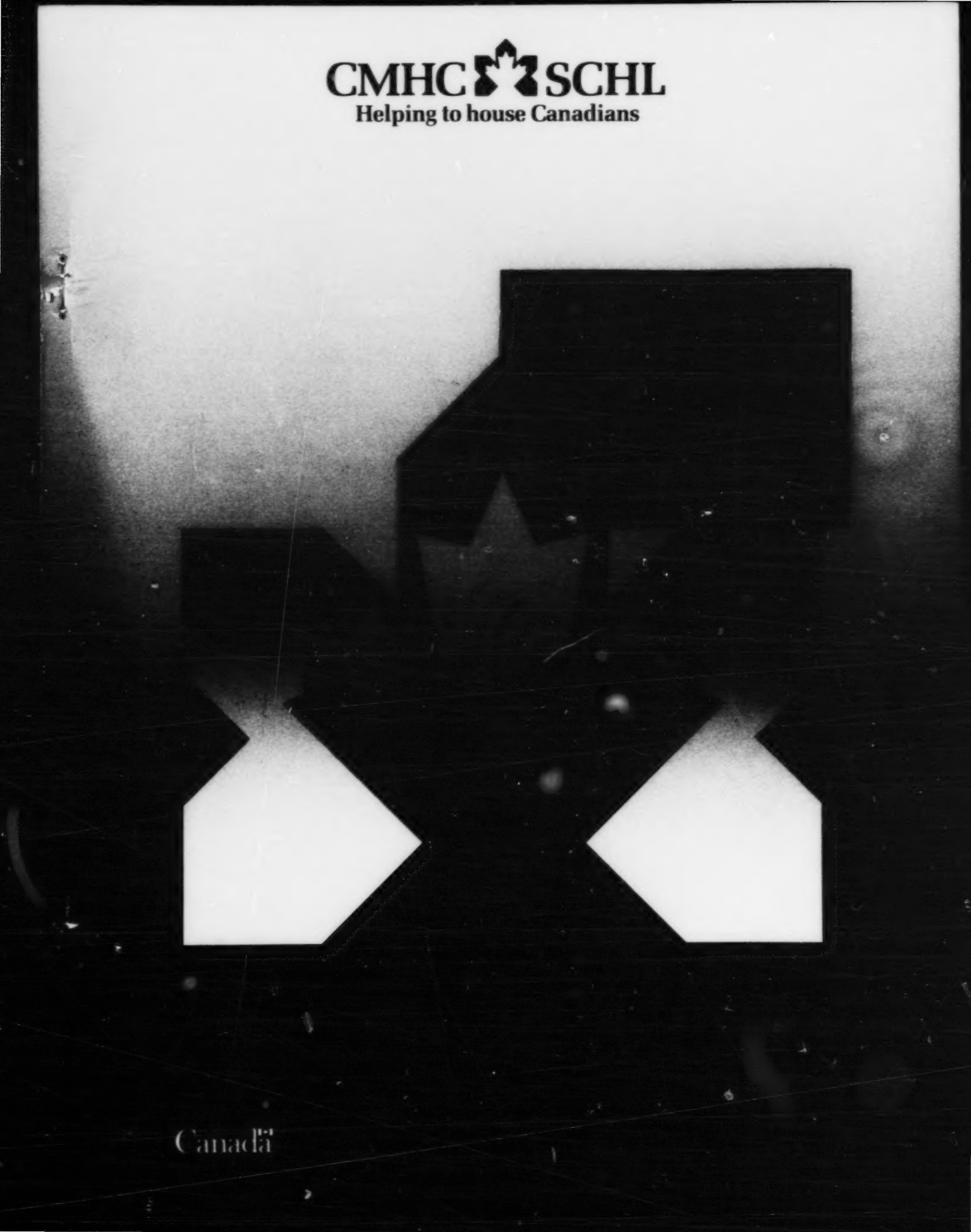
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JOIST ENHANCER

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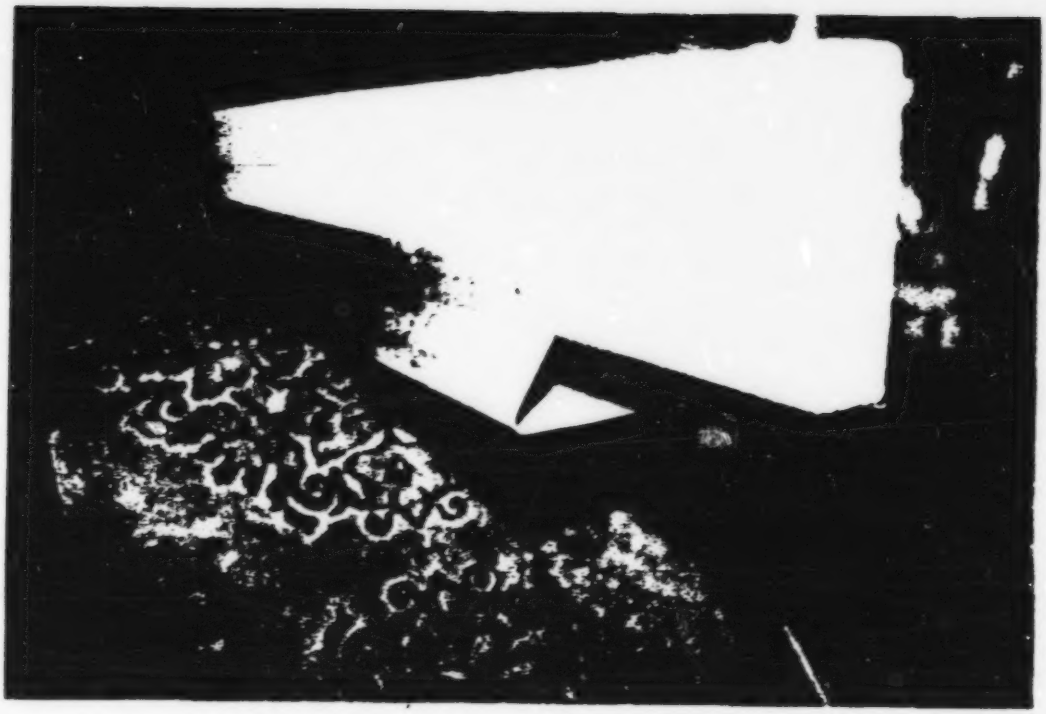
This project was carried out with the assistance of a grant from Canada Mortgage and Housing Corporation under the terms of the Housing Technology Incentives Program (HTIP). The views expressed are those of the author and do not represent the official views of the Corporation.



Project Title: JOIST ENHANCER

Author's Name: JAN JANOSIK, M. ENG, P. ENG

Report Completed: JUNE 29, 1990

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Producing this project was a group effort. Each member provided a pivotal skill that deserves my appreciation and acknowledgement. My thanks to Dr. Robert Drysdale for his valuable assistance and helpful criticism in devising and executing the testing program. Also thanks to Tom Redmond, P. Eng. who maintained the proper momentum and coordination necessary to keep this project in perspective and on time. Of course this project would not even exist without Canada Housing and Mortgage Corporation, Housing Technology Incentive Program which provided partial funding. In addition, the staff at CMHC were extremely helpful and cooperative for which I am thankful.

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- TEST REPORT ON BENDING BEHAVIOUR OF DUO-JOISTS

A. ABSTRACT

A new approach to reinforcing wood joists and existing wood flooring systems is examined. Specially formed sheet metal plates are fastened to the bottom of wood joists to enhance their stiffness and load carrying ability through composite action. The method is simple and inexpensive when compared to current methods for strengthening existing floors. It is also adaptable to new construction where longer spans and/or less vibration is desirable.

In this study a mathematical model was developed to predict the behaviour of the reinforced joist (DUO - JOIST) based on the widely used non-homogeneous beam analysis. The mathematical model was used to obtain:

1. An optimum shape and size of sheet metal plate (joist enhancer)
2. Most effective position of the joist enhancer in relation to the wood joist
3. Effect of the joist enhancer on different sizes of wood joist

Prototypes of the DUO - JOISTS were fabricated and tested to confirm the results of the mathematical model. Test results demonstrate the DUO - JOIST is effective in strengthening wood joists.

BACKGROUND

1. Description

A cold formed sheet metal joist enhancer in general channel cross section is bonded to the underside of a wood joist to form DUO - JOIST. The metal joist enhancer provides extra strength and stiffness to the wood joist without additional support beams and columns. The DUO - JOIST provide a straight side to attach a ceiling such as gypsum boards.

2. Benefit in existing buildings

In renovation projects one often finds that the floor joist are not adequate to carry the proposed loads , the deflection is beyond acceptable level , or the floor is too springy. This may happen in new houses as well.

One must provide additional support in the form of beam and columns, or, if there is room, it is possible to double up the joists. Additional strength can be drawn from the subfloor if an effective T section is achieved. Theoretically, the T section works, however, practical experience proves that the T section does not produce the desired results. Wood joist stiffening is costly and time consuming.

With the DUO - JOIST system, the sheet metal enhancer is attached to the underside of the existing wood joists. The result is a stronger stiffer floor system without losing any appreciable headroom or clear span. In addition, the metal enhancer provides a smooth level surface for attaching drywall or other finishing materials.

3.Benefit to new buildings

Although the above description implies that the joist enhancer is to be used in renovation, conversion or improvement projects, there is a wide range of applications for the DUO - JOIST in new house construction. Using the DUO - JOIST system, fewer wood joists are needed and longer spans can be achieved without sacrificing structural integrity or headroom. In the truss industry the cord members may be made of an inferior grade of wood and be of smaller size yet the finished product will have superior performance characteristics than those made today. Application of the joist enhancer to the top of the DUO - JOIST permits use of sheet metal decking in the floor or roof assembly. The DUO - JOIST system may even replace OWSJ's in short and medium span lengths.

C. OBJECTIVES OF THE PROJECT

The overall objectives of this preliminary study were as follows:

1. Develop a mathematical model to predict the behaviour of a wood joist reinforced with joist enhancer.

2. Use the mathematical model developed in step 1 to study different geometric shapes and configurations of joist enhancer to determine an optimal theoretical shape for testing.

3. Fabricate a number of prototype joist enhancers using the optimal theoretical shape(s) determined from step 2 and attach them to wood joists to form DUO - JOISTS in preparation for testing.

4. Design a simple testing procedure and test the DUO - JOIST (developed in step 3). Compare actual test results to the theoretical.

5. Determine areas for future study and improvement if the testing program provides positive results.

DETAILED DESCRIPTION OF PROTOTYPE DEVELOPMENT

1. Mathematical Model

The mathematical model was developed assuming composite action between the steel joist enhancer and the wood joist. Composite action is developed when two or more load- carrying structural members usually of different properties are integrally connected and act as a single unit under load. Structural members composed of steel and concrete are routinely designed for composite action and the design theories are well known.

The section properties for the DUO - JOIST were determined using the transformed area method. This method assumes:

a. Both materials remain in the elastic range under service loads.

b. The amount of load carried by each material is proportional to their relative strength as measured by the Young's Modulus (E).

For this study the steel was transformed into an equivalent area of wood using a modular ratio (E steel/E wood) varying from 18.288 to 20.828 depending on the type of wood studied (i. e. Select structural, No. 1, No. 2, or No. 3).

Three distinct shapes of joist enhancer were studied as outlined in the design notes contained in Appendix 1. A computer program was developed to compute section properties and maximum allowable joist spans while varying the following:

a. Thickness of metal used in each joist enhancer.

b. Size of wood joist.

c. Grade of wood.

d. Position of enhancer in relation to the wood joist.

2. Results of the model study

Analysis of results from the computer model yielded a number of interesting conclusions which are listed below in point form. (The complete study is contained in Appendix 1)

a. Increasing the gauge of metal used for the joist enhancer does not significantly increase the strength of the enhanced joist.For this reason 20 gauge metal was selected for the joist enhancer prototype since 22 gauge was thought to be too light to retain it's shape when loaded with drywall.

b. There was little difference in the increased allowable spans achieved by any of the three joist enhancer cross sections studied. As a result the "U" & ME shaped cross - sections were selected for prototype fabrication since these were seen as the most easily fabricated.

c. The percentage of strength increase calculated was greatest in the smaller joists (38 x 89) decreasing as depth of wood joist was increased. For this reason it was decided that 2" x 6" and 2" x 4" lumber would be used in the testing program since this is the smallest joist size normally used in standard construction.

The only unresolved issue after analysis was the number and type of fasteners to be used to join the joist enhancer to the wood joist. As there is no reliable way of determining the fastener spacing required to develop full shear transfer between the two materials (and therefore have full composite action), it was decided that a number of different combinations of nails and screws would be tested. From this empirical approach the most satisfactory bonding method would be selected for further testing and development.

3. Prototype Fabrication

The joist enhancer was fabricated by Riddell Sheet Metal using a 12 foot long brake and press / punch.

Wood joists were purchased from Turkstra Lumber and all wood was No. 1 or No. 2 Spruce.

DUO - JOIST prototypes were assembled using both ME and U shaped joist enhancers. Details on the species grouping of the wood and shape of enhancer used for each DUO - JOIST can be found in Table 1. The connection details between the wood joist and enhancer are summed up in Table 2.

Table 1: Summary of Test Specimens and Type of Tests

Test No.	Joist No.	Wood$^{\Delta}$ Type	Wood Section (mm × mm)	Overlap of Enhancer and Wood (mm)	Enhancer Type	Load Type
1	1	#2	38 × 138	38	ME	Static
2	2	#2	37 × 138	38	ME	Static
3	3	#2	38 × 140	38	ME	5 Cycles
4	3	#2	38 × 140		Reference	Static
5	4	#1	38 × 140	38	ME	Static
6	5	#2	38 × 140	38	U	Static
7	5	#2	38 × 140		Reference	Static
8	6	#2	38 × 140	38	U	Static
9	6	#2	38 × 140		Reference	Static
10	7	#1	38 × 138	37	U	Static
11	8	#1	37 × 141	37	U	2 Cycles
12	8	#1	37 × 141		Reference	Static
13	9	#1	38 × 137	37	U	Static
14	9	#1	38 × 137		Reference	Static
15	10	#2	38 × 138	38	U	5 Cycles
16	11	#1	38 × 140	38	U	Static
17	12	#1	40 × 89	*	U	2 Cycles
18	12	#1	40 × 89		Reference	Static
19	6	#2	38 × 140	38	**	2 Cycles
20	9	#1	38 × 138	37	**	Static
21	12	#1	40 × 89	39	**	Static

NOTE:

Joist No. -- See Table 2

ME & U -- See Fig. 4

Reference -- With enhancer removed

* -- 26 mm at one end, 54 mm at middle, 19 mm at the other end

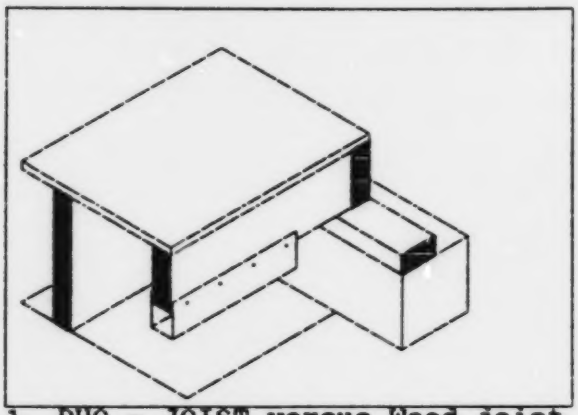
** -- Type U enhancer but bent up 30° from horizontal and 250 mm from the ends. For Test 19, the flanges of the enhancer were weakened by cuts of 50 × 89 mm at the mid-span, and cuts of 38 × 38 mm spaced 305 mm apart. For Test 20, the enhancer was spliced at 750, 1050, and 3050 mm from the left end of the enhancer with overlap length of 90 mm.

Δ #1 or #2 Wood Type refers to spruce species of #1 grade or #2 grade

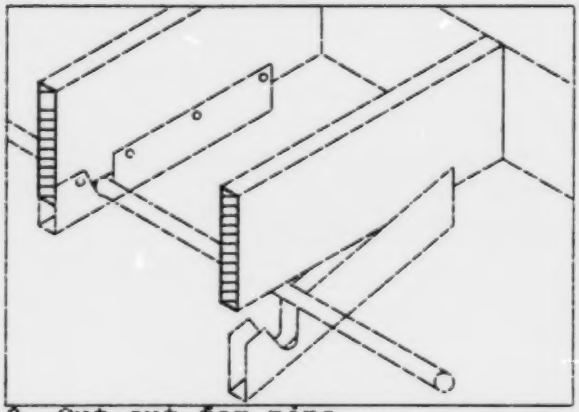
Table 2: Details of Connection Between Wood Joist and Enhancer

BOND TYPE	JOIST NO.
Four nails at end, then the rest of nails at 305 mm o/c	1
Four nails at end, then the rest of nails at 305 mm o/c	2
Two screws at 76 mm o/c, and this pair at 305 mm apart	3
Two screws at 76 mm o/c, and this pair at 305 mm apart	4
Three screws at end; 2 screws and wood block at mid-span (King's Post)	5
Four screws at end: 2 screws and block at 600 mm from ends (Queen's Post)	6
Continuous 1 = screws at 76 mm o/c entire span	7
Continuous 2 = screws at 305 mm o/c	8
Continuous 2 = screws at 305 mm o/c	9
Continuous 3 = ring nails at 76 mm o/c	10
Gang nail at 305 mm o/c = 19 mm cross	11
Continuous 2 = screws at 305 mm o/c with 25 mm pre-bow	12

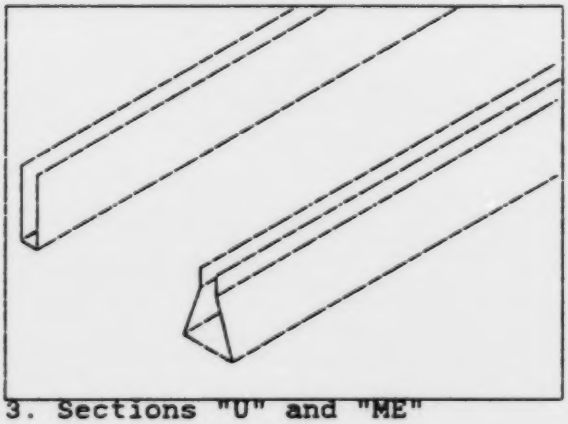
4. Drawings



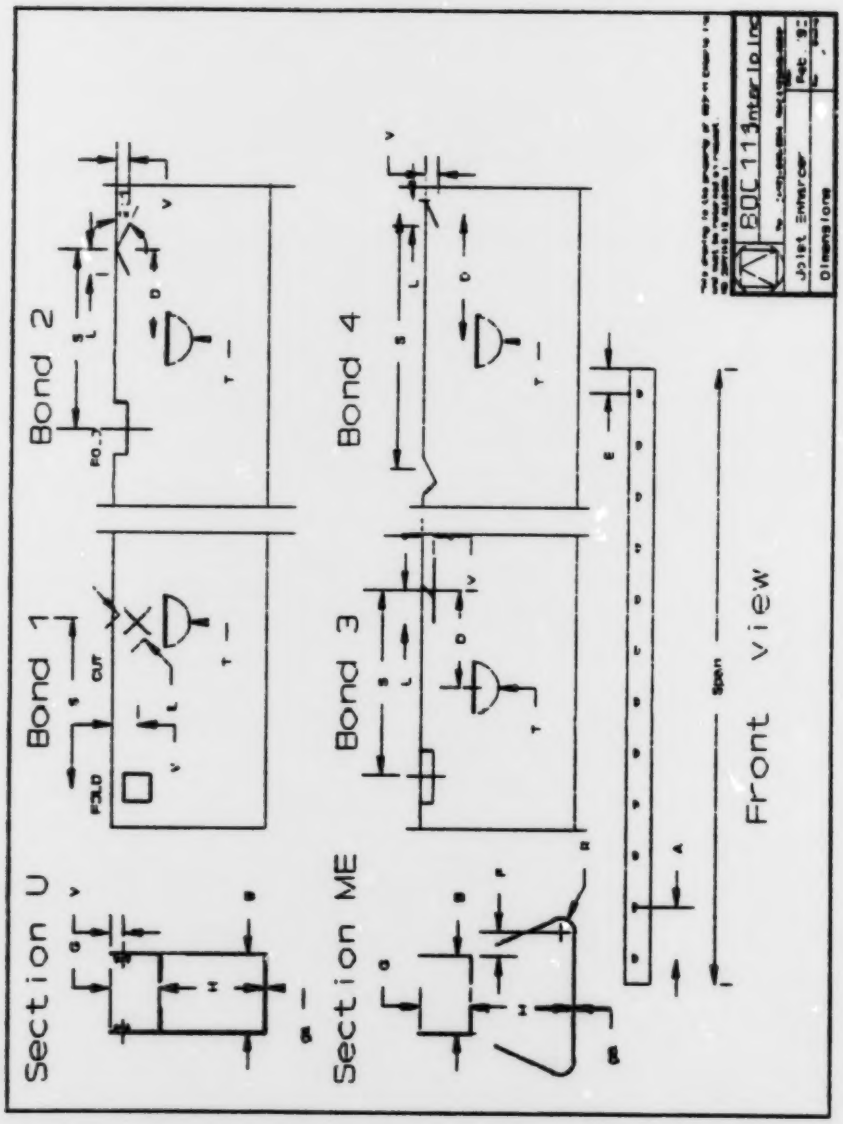
1. DUO - JOIST versus Wood joist



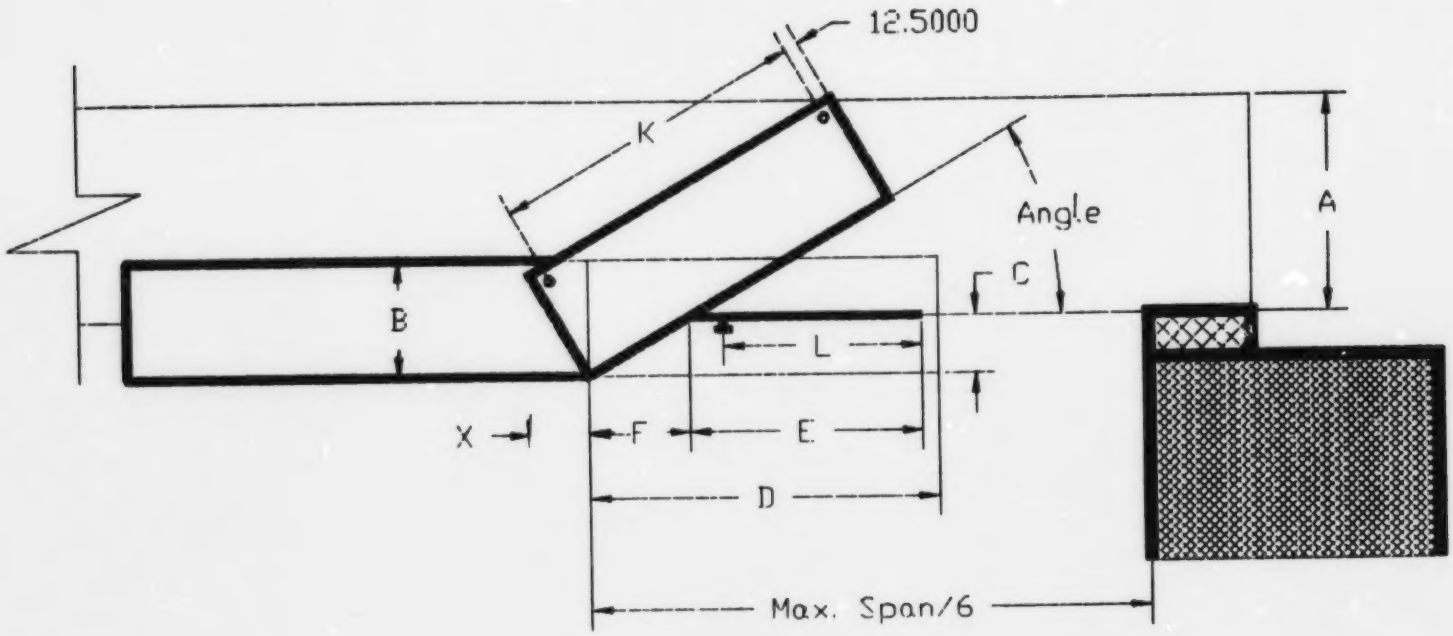
2. Cut-out for pipe



3. Sections "U" and "ME"



. Details of joist enhancers



DETAIL OF END ANCHORAGE

E. TESTING

For complete details of the testing program see "Test report on bending behaviour of DUO - JOISTS" contained in Appendix 2.

In order to assist in reviewing the test results the following tables and graphs were prepared for each DUO - JOIST.

Notes to Tables:

Column 1- calculated theoretical load to produce noted deflection in the DUO - JOIST as predicted by the mathematical model using E wood assumed. E being Young's Modulus of Elasticity.

Column 2 - actual test load to produce noted deflection in the DUO - JOIST as listed in the Table 3 of the appended report.

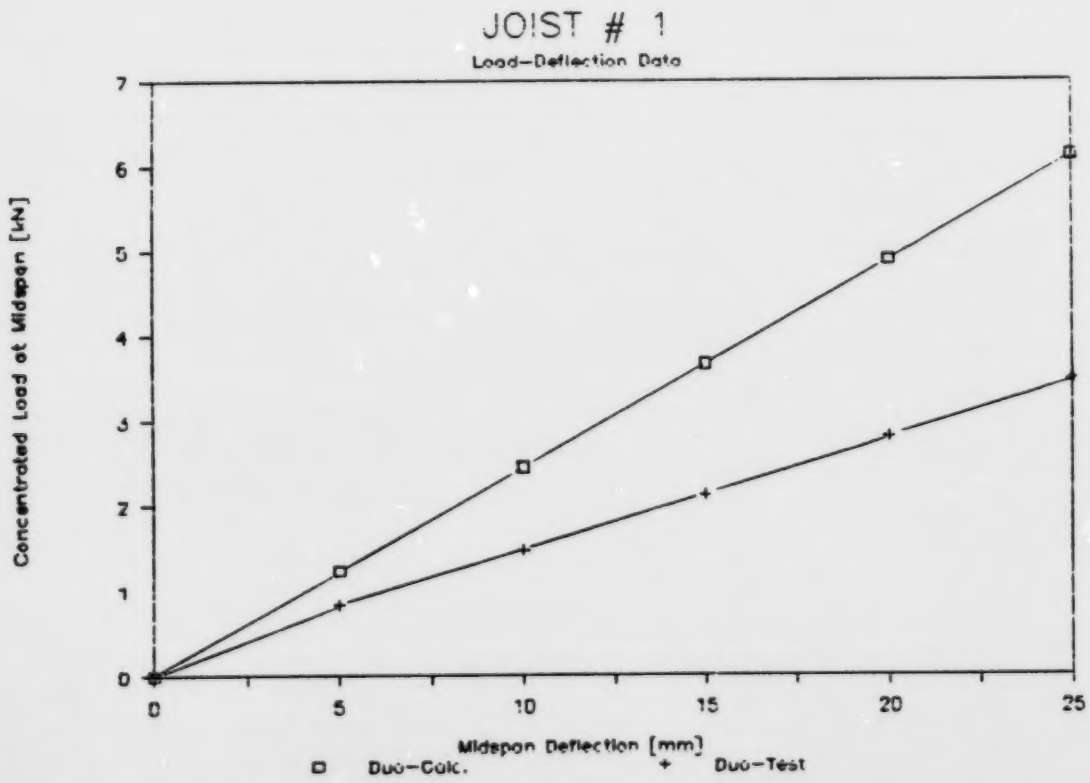
Column 3 - test load to produce noted deflection in the wood joist without enhancer where such reference test values were done.

Column 4 - test load to produce noted deflection in the DUO - JOIST with additional end anchorage.

JOIST # 1

Defl. [mm]	LOAD [kN]				Ew
	DUO JOIST		WOOD JOIST	ANCHOR	
	Calculated	Test	Test	Test	Calculated [Mpa]
0	0.000	0.000	0.000	0.000	
5	1.222	0.841			
10	2.443	1.470			0.00
15	3.660	2.118			
20	4.887	2.795			
25	6.108	3.478			

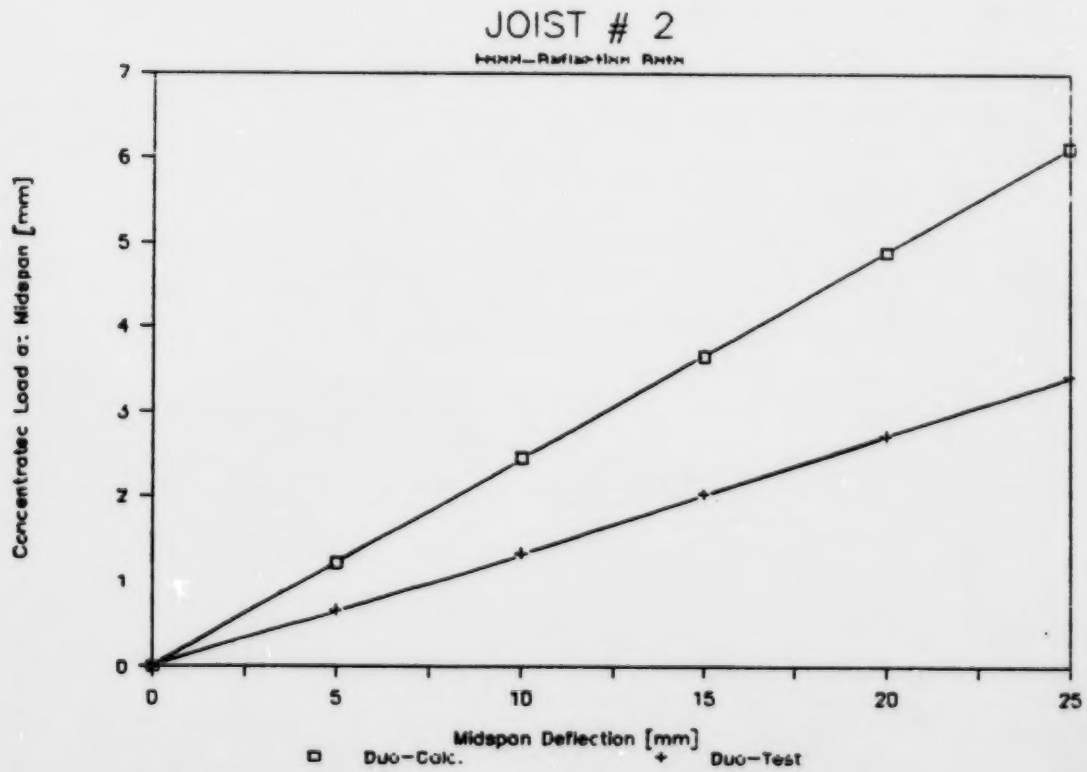
Notes:
Joist size: 38x138 Wood grade = #2
Enhancer type: ME
Bond: Four nails at the end, then the rest of nails at 305 mm o/c
Ewood assumed 8400 Mpa
Moment of Inertia (Duo Joist) = 3.35 E7
Moment of Inertia (Wood Joist) = 8.32 E6



JOIST # 2

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000	0.000	0.000	
5	1.221	0.664			
10	2.443	1.329			0.00
15	3.665	2.024			
20	4.887	2.723			
25	6.108	3.422			

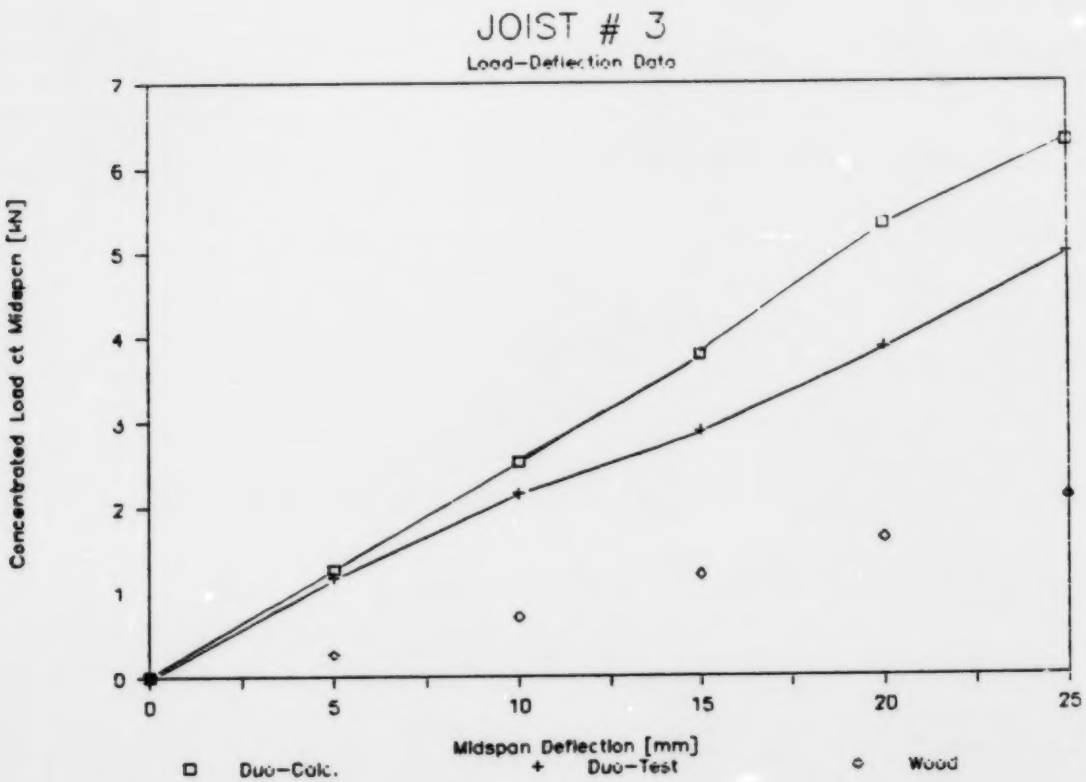
Notes:
Joist size: 37x138 Wood grade = #2
Enhancer type: ME
Bond: Four nails at end, them the rest of nails at 305 mm o/c
Ewood assumed 8400
Moment of Inertia (Duo Joist) = 3.45 E7
Moment of Inertia (Wood Joist) = 8.68 E6



JOIST # 3

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000	0.000	0.000	
5	1.258	1.159	0.254		
10	2.516	2.140	0.684		0.00
15	3.774	2.871	1.178		
20	5.320	3.866	1.616		
25	6.290	4.976	2.090		

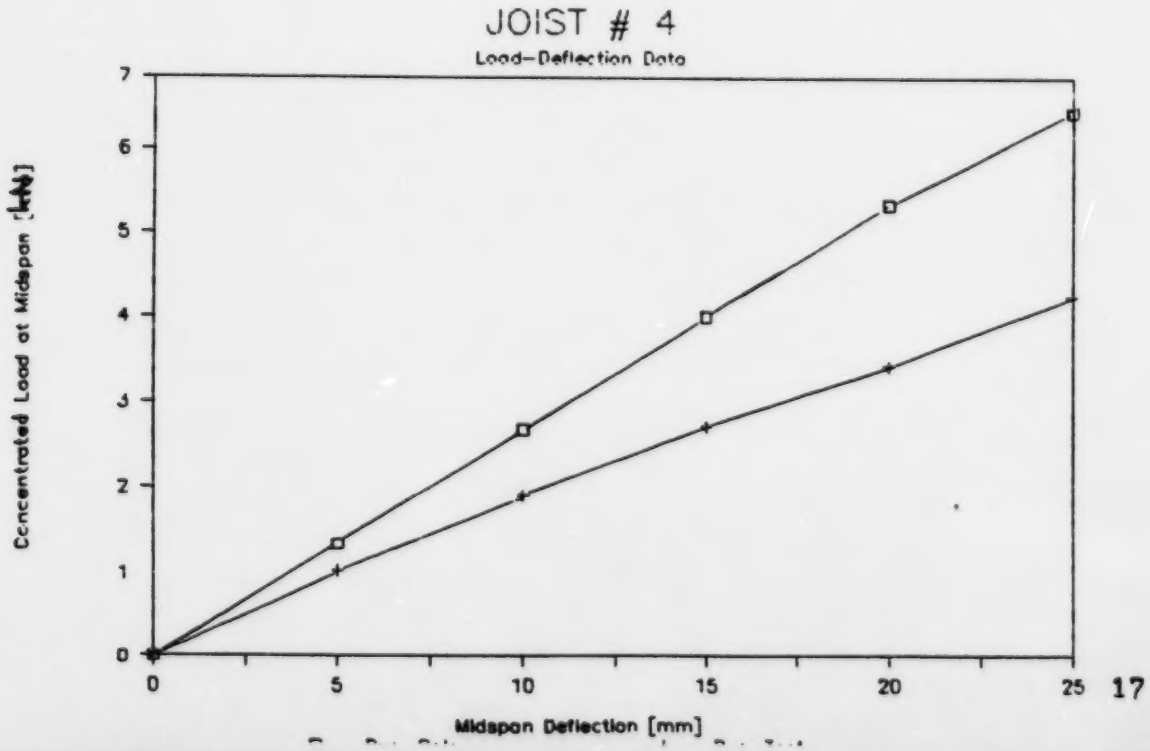
Notes:
Joist size: 38x140 Wood grade = #2
Enhancer type: ME
Bond: Two serews at 76 mm O/c, and this pair 305 mm apart
Ewood assumed 8400
Moment of Inertia (Duo Joist) = 3.45 E7
Moment of Inertia (Wood Joist) = 8.68 E6



JOIST # 4

Defl. [mm]	LOAD [kN]				Ew
	DUO JOIST		WOOD JOIST	ANCHOR	
	Calculated	Test	Test	Test	Calculated [Mpa]
0	0.000	0.000			
5	1.332	1.025			
10	2.664	1.901			0.00
15	3.996	2.694			
20	5.328	3.426			
25	6.600	4.250			

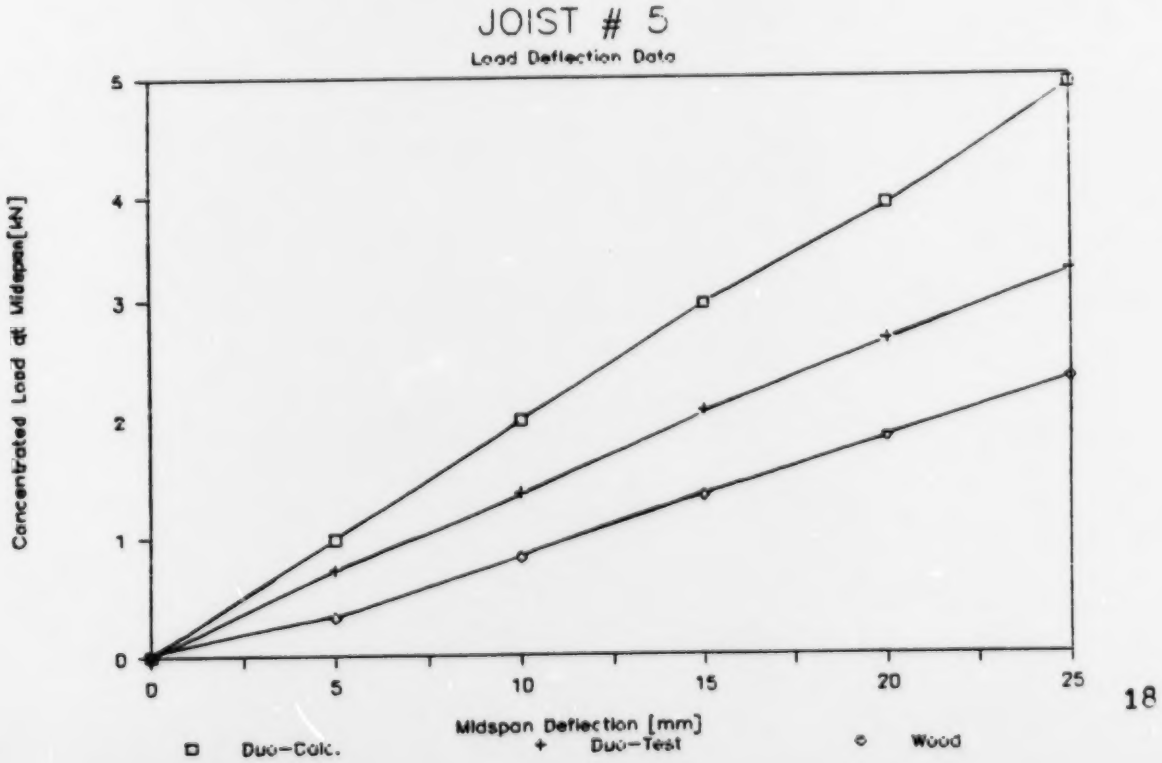
Notes:
Joist size: 38x140 Wood grade = #1
Enhancer type: ME
Bond: Two serews at 76 mm o/c, and this pair 305 mm apart
Ewood assumed 9300
Moment of Inertia (Duo Joist) = 3.3 E7
Moment of Inertia (Wood Joist) = 8.68 E6



JOIST # 5

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000	0.000		0.00
5	0.982	0.722	0.321		9584.00
10	1.965	1.361	0.821		10898.26
15	2.948	2.056	1.336		11823.04
20	3.931	2.647	1.818		12066.40
25	4.913	3.231	2.310		12265.52

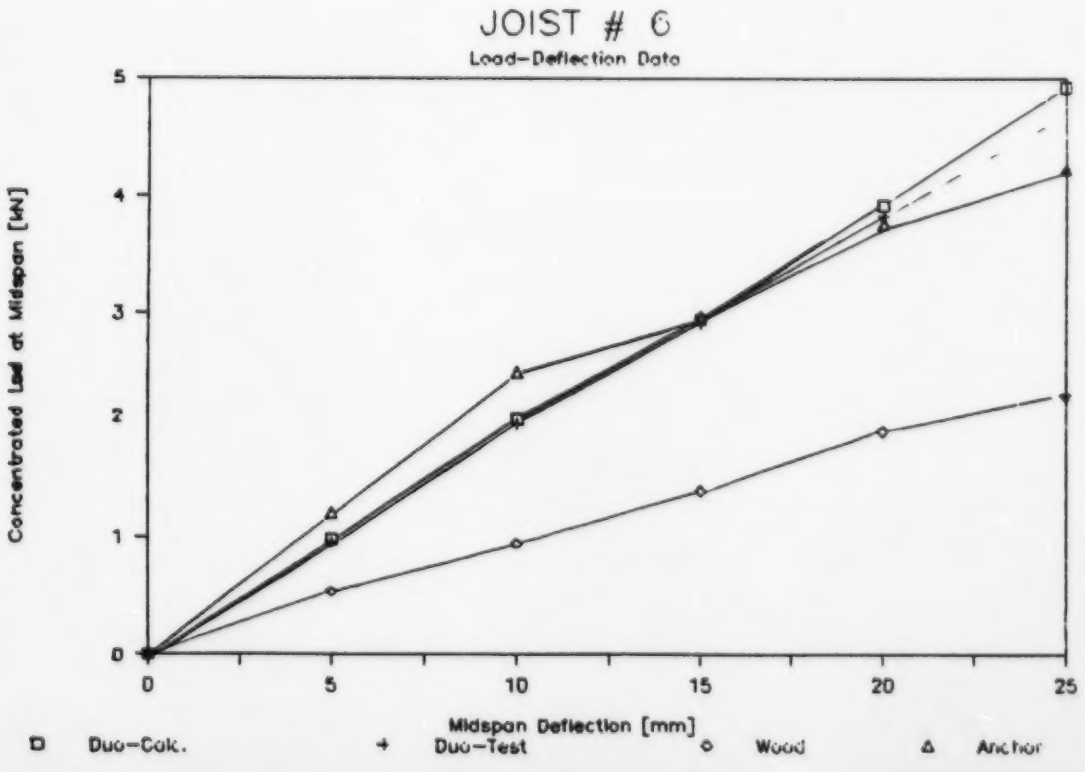
Notes:
Joist size: 38x140 Wood grade = #2
Enhancer type: U
Bond: Three screws at end; 2 screws and wood block at mid span.
Ewood assumed 8400 King's Post
Moment of Inertia (Duo Joist) = 2.696 E7
Moment of Inertia (Wood Joist) = 8.68 E6



JOIST # 6

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000	0.000	0.000	0.00
5	0.982	0.960	0.543	1.200	14415.96
10	1.965	1.933	0.943	2.500	12517.73
15	2.948	2.908	1.385	2.980	12256.67
20	3.931	3.827	1.886	3.770	12517.73
25	4.913		2.192	4.225	11638.97

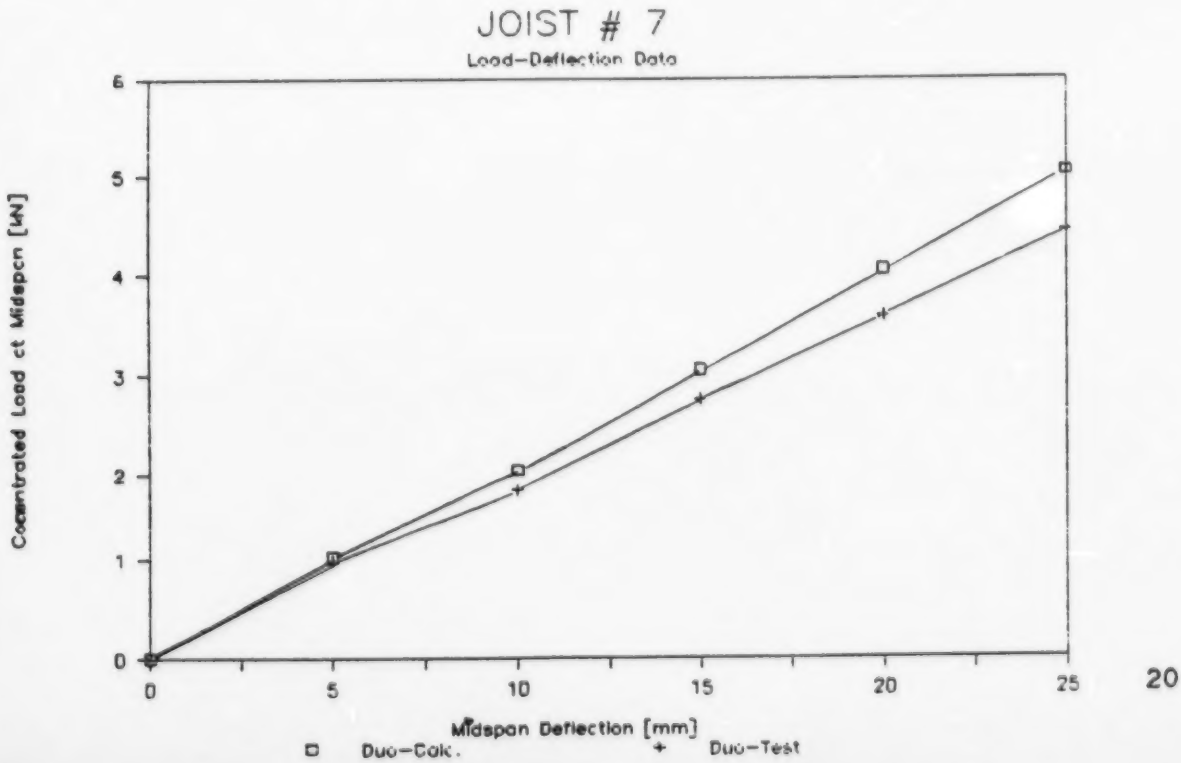
Notes:
Joist size: 38x140 Wood grade = #2
Enhancer type: U
Bond: Four screws at end; 2 screws & wood block at 600 from ends.
Ewood assumed 8400 Queen's Post
Moment of Inertia (Duo Joist) = 2.69 E7
Moment of Inertia (Wood Joist) = 8.68 E6



JOIST # 7

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000			
5	1.010	0.984			
10	2.020	1.831			
15	3.030	2.736			
20	4.040	3.584			
25	5.051	4.445			

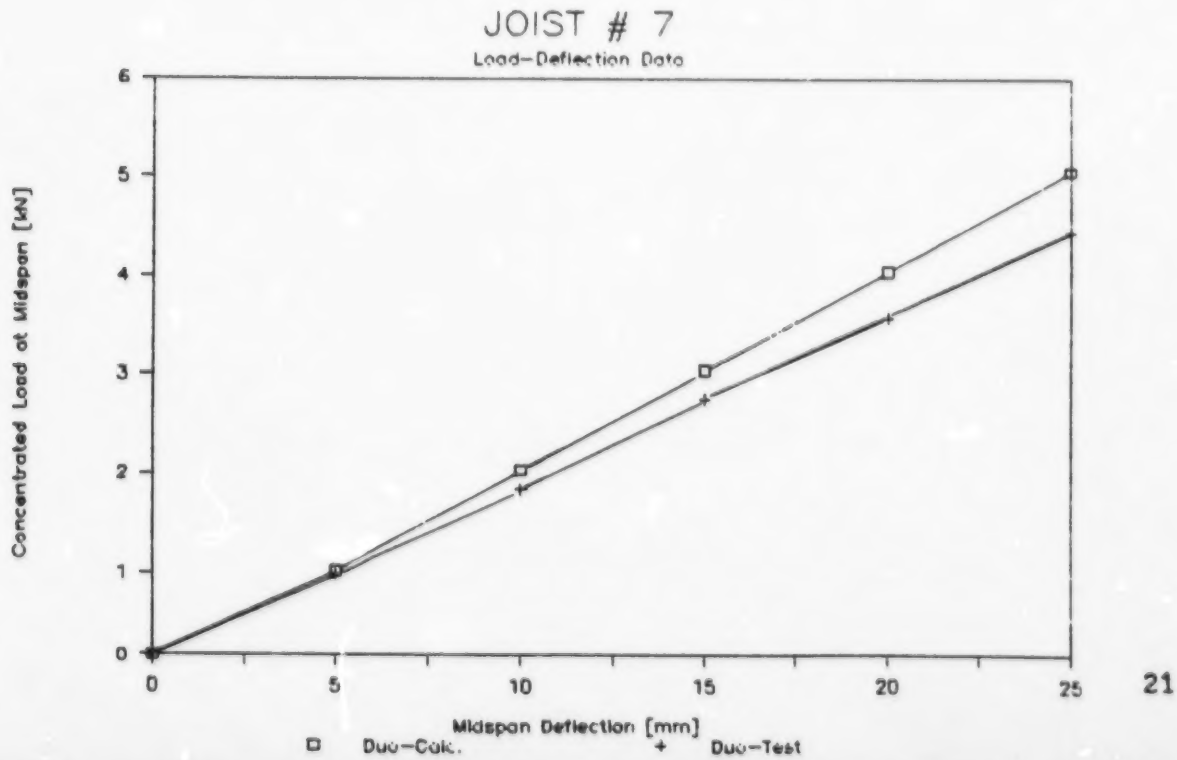
Notes:
Joist size: 38x138 Wood grade = #1
Enhancer type: U
Bond: Continuous 1 = screws at 76 mm o/c entire span.
Ewood assumed 9300
Moment of Inertia (Duo Joist) = 2.5 E7
Moment of Inertia (Wood Joist) = 8.32 E6



JOIST # 8

Defl. [mm]	LOAD [kN]				Ew
	DUO JOIST		WOOD JOIST	ANCHOR	
	Calculated	Test	Test	Test	Calculated [Mpa]
0	0.000	0.000	0.000		
5	1.060	0.831	0.253		6512.95
10	2.121	1.552	0.678		8807.24
15	3.182	2.240	1.139		9863.76
20	4.242	2.881	1.528		9924.38
25	5.303	3.545	1.887		9804.87

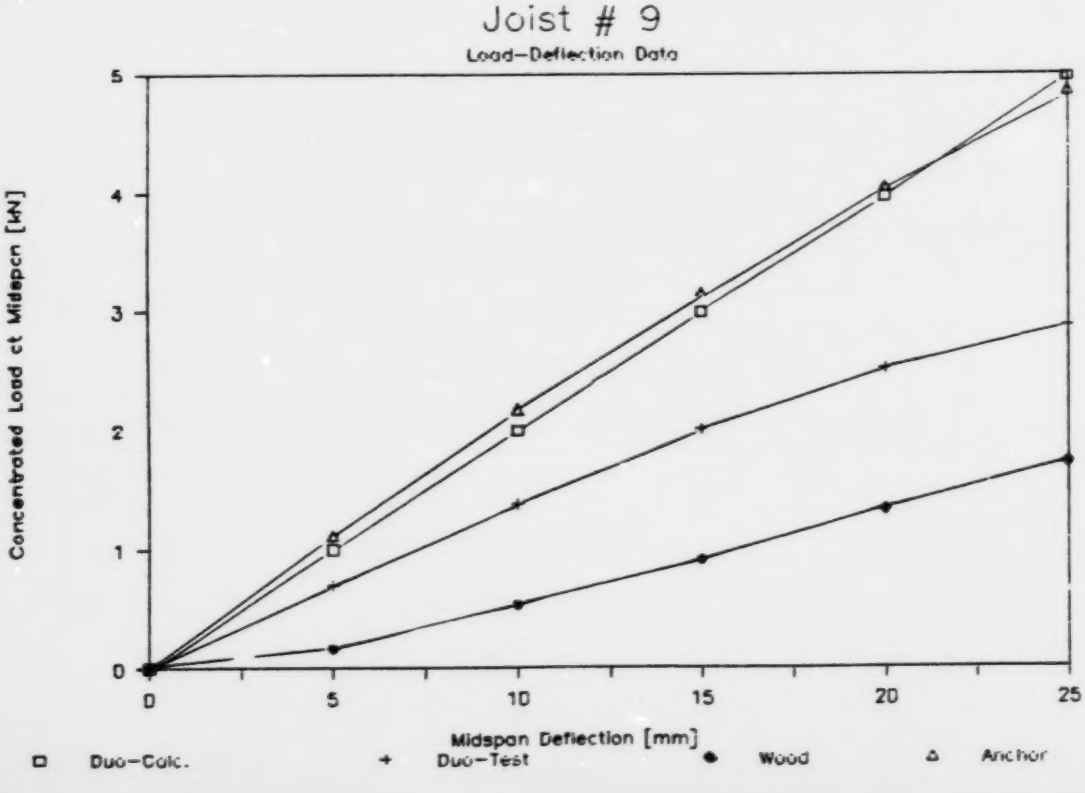
Notes:
Joist size: 37 x 141 Wood grade = #1
Enhancer type: U
Bond: Continuous 2 = screws at 305 mm o/c.
Ewood assumed 9300
Moment of Inertia (Duo Joist) = 2.62 E7
Moment of Inertia (Wood Joist) = 8.87 E6



JOIST # 9

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000	0.000	0.000	0.00
5	0.993	0.694	0.164	1.110	4642.83
10	1.987	1.375	0.530	2.160	7502.14
15	2.981	1.988	0.903	3.150	8521.29
20	3.975	2.507	1.336	4.050	9455.52
25	4.968	2.864	1.726	4.850	9772.60

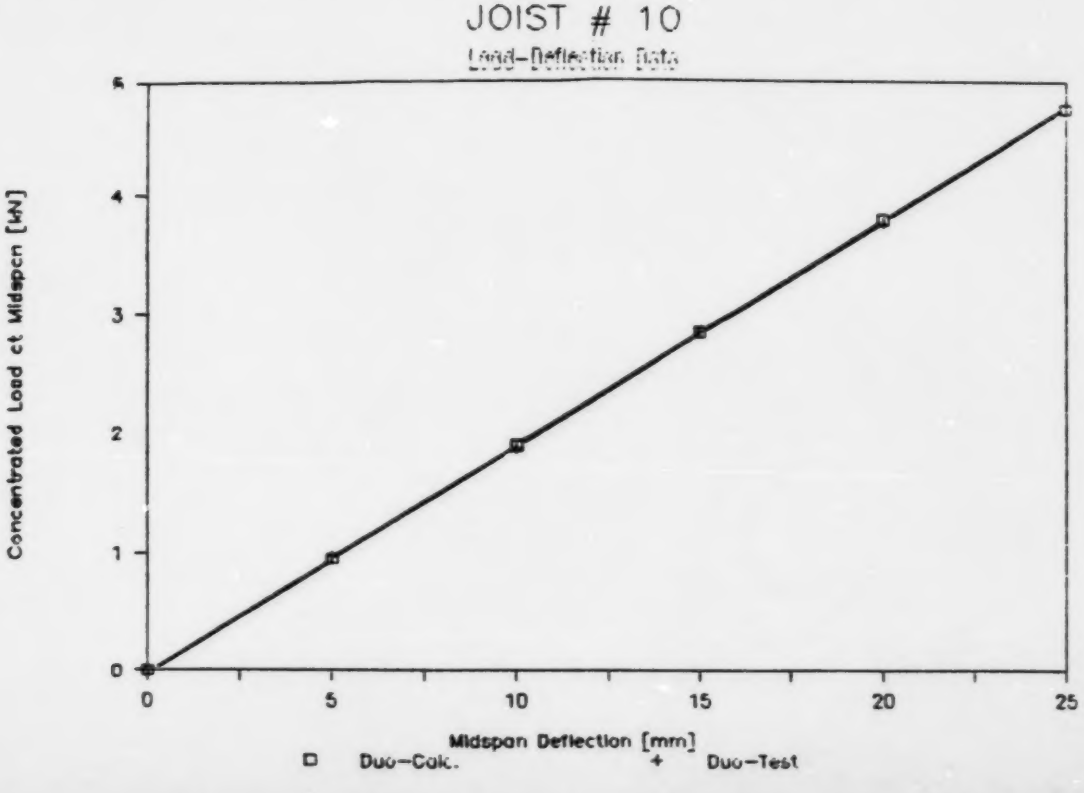
Notes:
Joist size: 38 x 137 Wood grade = #1
Enhancer type: U
Bond: Continuous 2 = screws at 305 mm o/c.
Ewood assumed 9300
Moment of Inertia (Duo Joist) = 2.46 E7
Moment of Inertia (Wood Joist) = 8.14 E6



JOIST # 10

Defl. [mm]	LOAD [kN] DUO JOIST Calculated	DUO JOIST Test	WOOD JOIST Test	ANCHOR Test	Ew Calculated [Mpa]
0	0.000	0.000			
5	0.951	0.976			
10	1.902	1.882			
15	2.854	2.858			
20	3.805	3.782			
25	4.756	4.712			

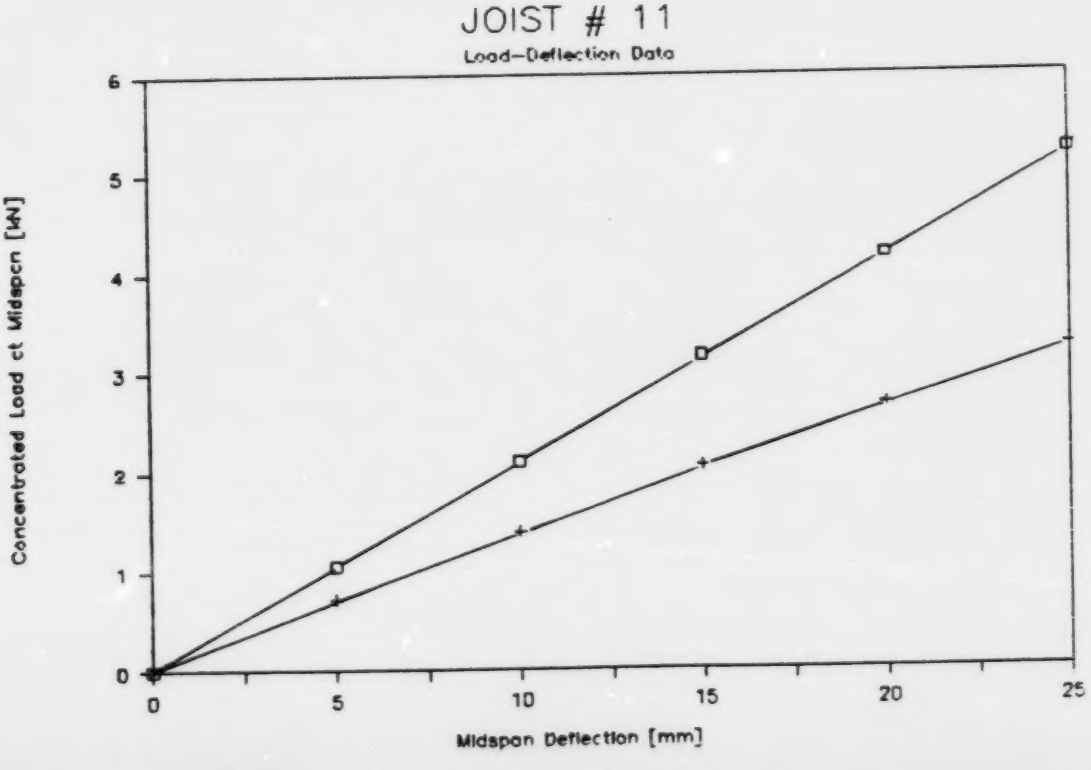
Notes:
Joist size: 38 x 137 Wood grade = #2
Enhancer type: U
Bond: Continuous 3 = ring nails at 76 mm o/c.
Ewood assumed 8400
Moment of Inertia (Duo Joist) = 2.6 E7
Moment of Inertia (Wood Joist) = 8.32 E6



JOIST # 11

Defl. [mm]	LOAD [kN]				Ew
	DUO JOIST		WOOD JOIST	ANCHOR	
	Calculated	Test	Test	Test	Calculated [Mpa]
0	0.000	0.000			
5	1.043	0.714			
10	2.087	1.387			
15	3.137	2.042			
20	4.174	2.661			
25	5.218	3.233			

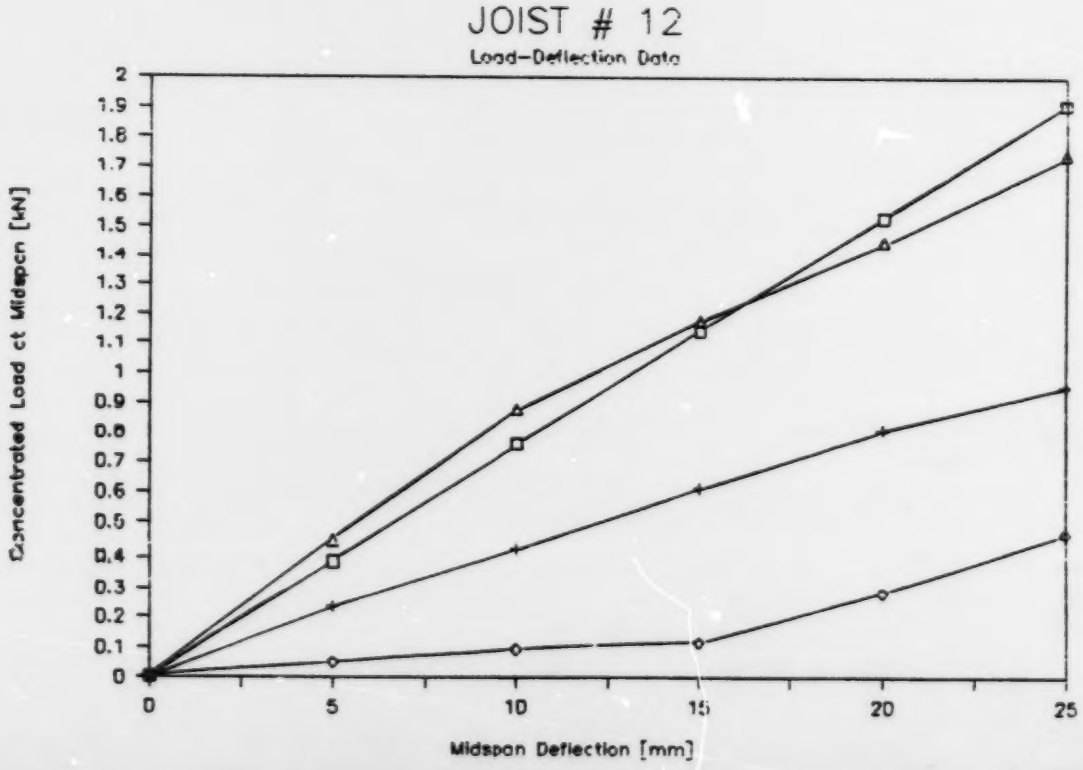
Notes:
Joist size: 38 x 140 Wood grade = #1
Enhancer type: U
Bond: Gang nail at 305 mm o/c = 19 mm cross.
Ewood assumed 9300
Moment of Inertia (Duo Joist) = 2.58 E7
Moment of Inertia (Wood Joist) = 8.68 E6



JOIST #12

Defl. [mm]	LOAD [kN]				Ew
	DUO JOIST		WOOD JOIST	ANCHOR	
	Calculated	Test	Test	Test	Calculated [Mpa]
0	0.000	0.000	0.000	0.000	0.00
5.00	0.381	0.237	0.047	0.445	4856.87
10	0.762	0.425	0.094	0.880	4856.87
15	1.144	0.614	0.116	1.180	5718.02
20	1.525	0.813	0.283	1.450	7311.14
25	1.907	0.961	0.469	1.740	9693.07

Notes:
Joist size: 40 x 89 Wood grade = #1
Enhancer type: U
Bond: Continuous 2 = screws at 305 mm o/c with 25 mm pre-bow.
Ewood assumed 8400
Moment of Inertia (Duo Joist) = 1.46 E7
Moment of Inertia (Wood Joist) = 2.23 E6



F. ASSESSMENT AND COMMENTARY

The overall objectives of this project have been met.

A mathematical model has been developed to predict the behaviour of the DUO - JOIST system and it appears to approximate the DUO - JOIST behaviour if sufficient anchorage and bonding is provided to allow full composite action to develop.

In fabricating the DUO - JOISTS it was discovered that:

1. Use of truss plate connections (Gang Nails) for bonding the enhancer to the wood joist is not practical without the use of specialized equipment. It is difficult to position the enhancer on the wood joist and set the teeth into the wood. The truss plate teeth protrude into the space where the wood joist will slide making it necessary to spread the enhancer legs which is difficult and awkward.

2. The maximum size of brake for forming the joist enhancer is 12 feet. This limits the length of joist enhancer fabricated by the use of existing brakes to 12 feet. Longer spans would require rolling equipment to form the joist enhancer or splicing.

3. It is faster to install nails than screws and there is no appreciable strength increase when using screws.

4. Twenty gauge sheet metal is not easy to cut with ordinary snips. Experience with steel stud installations would indicate that 20 gauge or thicker steel should be cut using a saw. Twenty-two gauge is the practical cut-off point for cutting with snipper.

The following observations were made while testing:

1. The wood joists were wet when the joist enhancers were attached. As the DUO - JOISTS dried they warped and the test results may have been somewhat affected by the warpage.

2. Gang nail connections do not appear to be practical for use on wet lumber. As the wood dries the connections tend to become loose.

3. Only mid-span deflection was measured with any degree of accuracy. Although an attempt was made to measure movement of the joist enhancer at the ends it was not sufficiently reliable to include in this report.

4. No strain gauges were employed in this study to show the distribution of stresses along the DUO - JOIST. This was not seen as critical at this preliminary stage.

5. Differential movement between the wood joist and joist enhancer was not measured.

6. No DUO - JOISTS were tested to failure (ultimate load).

7. The possibility of residual stresses in the DUO - JOIST due to drying of the wood was not examined.

8. End anchorage appeared to be critical in developing full composite action.

CONCLUSIONS AND RECOMMENDATIONS

From the testing program the following conclusions were drawn:

1. Approximately twice as much load is required to produce 25 mm deflection in a joist using a joist enhancer when compared to an ordinary joist

2. Duo-Joists with screws 76 mm apart are about 30% stronger than those with screws 305 mm apart, and the flanges suffer less severe local buckling

3. The load-deflection behaviour of the Duo-Joists may be approximated as linear elastic

4. Prestressing is effective for reducing the net deflection, but the screws need to be more closely spaced to reduce the lateral buckling of the flanges of the enhancer

5. No significant difference was observed between the nailed or screwed connections

6. Load-deflection response is not significantly different for the Type ME and U enhancer, but the truss type connection (enhancer with additional end anchorage) appears to provide superior performance in bending

The results of the testing confirm that the DUO - JOIST system does enhance performance in a manner which can be approximated by the mathematical model developed in this study. This initial work although comprehensive cannot be considered conclusive. It lays the groundwork for future study in the following areas:

1. Additional laboratory testing of prototypes using larger wood joists and a more sophisticated testing procedure which will show strain distribution through the critical sections

2. Field installation to observe the long term effects of loading and shrinkage on the DUO - JOIST.

3. Further refinement of the bonding/end anchorage system for tying the joist enhancer to the wood joist in order to optimize composite action.

4. It may be possible to replace sawn lumber joists with plywood or other materials in order to enhance performance or decrease cost

5. An advanced study might include the fire performance and sound transmission qualities of an assembly using the DUO - JOISTS

A1 General Theory

The basic idea of non-homogeneous beam analysis are covered in standard texts on strength of materials (References 7,8,9 and 10).

Non-homogeneous beams are those in which material properties vary from point to point; in particular we consider a beam cross section of combination of materials with various elastic moduluses E(y) being function of the depth of the section. The strength of materials method conditions are as follows:

a/ Equilibrium

$$P=0 \quad : \quad \int F \, dA = 0 \qquad (a1)$$

$$Mx = 0 \quad : \quad \int y \, F \, dA = M \qquad (a2)$$

b/ Geometry (assuming that plane section remains plane)

$$= y \qquad (b)$$

c/ Stress-strain relations

$$F = E(y) \; = E(y) \; y \qquad (c)$$

Equation (c) indicates that the stress at any point is proportional to both the material stiffness and the distance from the neutral axis.

Substituting Equation (c) into Equations (a1) and (a2):

$$P = 0 \quad : \quad \int E(y) \; y \, dA = 0 \qquad (1)$$

$$M = 0 \quad : \quad \int E(y) \; y^{2} dA = 0 \qquad (2)$$

Equation (1) serves to locate the neutral axis, and the Equation (2) is used to find the curvature, which is resubmitted into Equation (c) to find the stress.

Stiffness variation of the cross section is expressed by

$$E = Ew \quad \text{for the wood}$$
$$E = Es \quad \text{for the steel} \qquad (3)$$

The geometry of the deformed structure is given by Equation (b) which together with the stress-strain relations (3) leads to the stress distribution

$$F = Ew \; = Ew \; y \quad \text{for the wood}$$
$$F = Es \; = Es \; y \quad \text{for the steel} \qquad (4)$$

and, when substituted into the first equilibrium Equation (a1) yields

$$P = 0 \quad : \quad \left(Ew \int y \, dA + Es \int y \, dA \right) = 0$$

or, using the modular ratio n=Es/Ew

$$Ew \left(\int y\, dA + n \int y\, dA \right) = 0 \qquad (5)$$

Since the factor Ew is non-zero , therefore the neutral axis must be so located that the factor in parenthesis vanishes. Hence,

$$\left(bw \int y\, dy + n\, bs \int y\, dy \right) = 0 \qquad (6)$$

$dA = bw\, dy$ $dA = bs\, dy$

The location of the neutral axis is found by solving Equation (6).

The quantity of

$$\left(\int y^2\, dA + (Es/Ew) \int y^2\, dA \right)$$ represents the moment of inertia of the transformed section It.

therefore

$$Ew\ It = M \qquad (7)$$

From the Equation (7) the curvature can be calculated for any moment M.

A2 Balanced design of joist reinforcement

It seems appropriate to proportion the section profile so that the allowable stress in each material will be reached at the same time, thus using both components as efficiently as possible taking into account the different strength of used materials.

Assume that the perfect bond exists between steel enhancer and wood joist. Furthermore, only the horizontal portion of steel is contributing to the strength of the section while the web portion is ignored (may be partly cut off due to location of service conduits running on the underside of joists).

Condition of Statics:

1. Section in bending has a neutral axis that divides compression and tension sides. The resultant internal compressive force = resultant internal tensile force.

2. Moment of internal couple = applied bending moment.

a/ Section profile b/ Strains c/ Stresses

Fig. 1 Composite beam

From the Fig. 1 b/ by similar triangles,

$$w/\ s = x / (d - x) \qquad (8)$$

and from Fig. 1 c/

$$C = 1/2\ b\ x\ Fw$$

$$T = Fs\ As$$

Using Equation(c) and Equation (3) and substituting into Equation (8) for appropriate material gives

$$Fw / Fs = x / [\ n\ (d-x)] \qquad (9)$$

The first condition of statics requires that the forces C and T be equal. That gives

$$C = T \ : \ 1/2\ b\ x\ Fw = Fs\ As$$

or $$Fw / Fc = 2\ As / (b\ x) \qquad (10)$$

The ratio Fw/Fs may be eliminated by setting Equation (9) equal to Equation (10); thus,

$$2\,As / (b\,x) = x / [\,n\,(d-x)]$$

or

$$x^2 + x\,(\,2\,As\,n/b) - (\,2\,As\,n/b) = 0$$

and the solution for this quadratic equation gives

$$x = -B + \sqrt{B^2 + 2B} \qquad (11)$$

where

$$B = 2\,As\,n\,/b$$

The joist is ideally reinforced when the situation is such that:

$$\text{Actual Fw / Actual Fs} = \text{Allowable Fw / Allowable Fs}$$

Using principles of an ideally reinforced concrete section then

$$k = \text{ideal } x / d = \text{allow Fw } /(\text{allow Fs}/n - \text{allow Fw})$$

$$j = 1 - k/3$$

$$p = k\,(\text{allow Fw}) / [2(\text{allow Fs})]$$

and

$$R=1/2\,(\text{allow Fw})\,j\,k$$

are similar to the well known " four design constants" of the working stress design method for reinforced concrete. These constants are listed in the Table 1.for S-P-F species and steel having yield stress of 60 ksi using values from Table 8 of CSA Standard CAN3-O86-M80 for the reasons of consistency with the Max. allowable spans listed in Ontario Building Code. Table 2 is based on the Reference(2) due to forthcoming changes in the Code.

Table 1. Design constants for S-P-F; CAN3-O86-M80

	Select	No.1	No.2	No.3
Ew	9300.000	9300.000	8400.000	7500.000
n=Es/Ew	21.828	21.828	24.643	27.067
Fw	8.900	7.700	6.200	3.600
k	0.516	0.480	0.456	0.349
j	0.828	0.840	0.848	0.884
p	0.013	0.010	0.008	0.003
R	1.902	1.553	1.199	0.555

Table 2. Design constants for S-P-F; CAN3-O86-M84

	Select	No.1	No.2	No.3
Ew	11100.000	10200.000	10200.000	9700.000
n	18.288	19.902	19.902	20.928
Fw	9.000	6.500	6.500	4.800
k	0.475	0.415	0.415	0.356
j	0.842	0.862	0.862	0.881
p	0.012	0.007	0.007	0.005
R	1.799	1.163	1.163	0.752

Fs=182.02 MPa (Fy = 60 ksi), Es = 203 000 MPa

A3. Design Examples

EXAMPLE 1.

Given: Bending Moment BM=2.16 kN-m
Wood joist 38x140 (2"x6") of S-P-F species , #1 grade using M80 Standard. Enhancer extends 50 below the joist.
Determine:
Required width of reinforcing enhancer.
****

The depth of the profile , d = 140 + 50 = 190mm
From Table 1. the design constants are:

Ew = 9,300 MPa	k = .48	R = 1.553 MPa
Fw = 7.7 MPa	j = .84	
n = 21.828	p = .01	

Resisting Moment of ideally reinforced 38x190 profile = $R\ b\ d^2$

$RM = 1.553 * 38 * 190^2 = 2,130,405$ N-mm = 2.13 kN-m

The 38x190 section if it is ideally reinforced, will not carry the given bending moment of 2.16 kN-m. The compressive force C must be increased so the neutral axis is lowered.
The solution to find actual position of neutral axis comes from the second condition of Statics:

$$M = C * arm = 1/2\ (Fw\ b\ x)(d - x/3)$$

$$2.16(10^6) = 1/2 *(\ 7.7 * 38 * x)(\ 190 - x/3)$$

or

$$48.76\ x^2 - 27,797,000\ x + 2.16(10^6) = 0$$

which yields to x = 92.809 mm
(ideal x = k d = .48 * 190 = 91.2 mm)

Actual C=1/2*7.7*38*92.809 = 13,577.95 N = 13.577 kN
Actual jd = 190 - 92.809/3 = 159.06 mm
R.M. = C * jd =2,159,759.58 N-mm = 2.159 kN-m (checks)
Actual Fs = n Fw (d - x)/x = 21.828*7.7*(190-92.809)/92.809 =
= 176.0113 MPa , which is less than allowable. That's penalty for lowering the neutral axis.

From first condition of Statics:

$$T = C \ : \ As\ Fs = C$$
$$As = C/Fs = 13,577.95 / 176.0113 = 77.1425\ mm^2$$

Using gauge 20 , the thickness of metal is .832mm
and therefore required width is As/.832 = 92.719mm

Flankage = (92.719 -38)/2 = 27.359 mm

EXAMPLE 2.

Given: BM = 2.16 kN-m Joist 38x140 #1 S-P-F; M80 Standard
with 100 mm enhancer extension

Determine:
Required width of the enhancer.

****

Use same design constants as in the Example 1.

d = 140 + 100 = 240 mm

R.M. = R b d^2 = 1.553 * 38 * 240 * 240 = 3.14 kN-m > 2.16 kN-m

This means that the section is underreinforced!
Area of steel can be determined from:

As = BM / (Fs j d) = 2.16(10^6) /(182.02 * .84 * 240) = 58.86 mm^2

Note: this Equation for determination of the amount of steel must never to be used for over reinforced section. Use deeper section to avoid overreinforcing.

Use 20 gauge with t=.832mm L = 58.86 / .832 = 70.75 mm

A4. TRANSFORMED SECTION

The following is the flow chart for the computer program to calculate neutral axis location, moment of inertia, and sectional modulus of the compression and tension zone of the reinforced profile.

INPUT

Wood: size B=Width of wood 38mm
DW=Depth of wood, mm
properties Ew= Modulus of Elasticity of wood, MPa
Fw= Allowable stress in extr. fibre, MPa
Steel:size H=Vertical extension of enhancer from bottom edge of joist, mm
HH=clip part of enhancer along the wood, mm
F =Flankage beyond width of joist, mm
T =Thickness of metal ,mm
R =Radius of bending in horizontal to vertical face of enhancer ,mm
properties Es=Modulus of Elasticity 203,000 MPa
Fs=Allow. stress in steel 182.02 MPa

MISCELLANEOUS

n=Es/Ew Transformation ratio
TH Transformed thickness of metal
D Depth of composite profile

ENHANCERS'S PROPERTIES

1. Geometry, Total length
2. Location of neutral axis measured to lower extreme.
3. Moment of Inertia to it's N.A.

PROPERTIES OF COMPOSITE PROFILE

1. Location of N.A. to the top of composite section
2. Moment of Inertia to N.A. of composite section
3. Sectional Modulus for tension and compression zone

PRINT ABOVE CALC. PROPERTIES

END

Program listing of LMAX for calculatin max. spans of joists with and without enhancer

```
10 REM LMAX program calculates maximum span of the joist
20 REM
30 REM
40 PI=3.14152654# :R=3 :H=50 :HH=25 :HHH=H+HH :HHHH=HHH-R
50 ES=203000! :TH=.906 :KH=1.15 :LL=1.9
60 REM
70 LPRINT"Wood-commercial designation S-P-F"
80 LPRINT"================================="
90 LPRINT"For uniform live load=";LL"kN/m^2"
100 REM
110 REM
120      FOR JJ=1 TO 4
130 READ GRADE$(JJ),FB(JJ),FBW(JJ),EW(JJ)
140      NEXT JJ
150      FOR II=1 TO 5
160 READ KZB(II)
170      NEXT II
180      FOR III=1 TO 5
190 READ HW(III)
200      NEXT III
210      FOR J=1 TO 4
220 LPRINT "GRADE = ";GRADE$(J)
230 REM Transform thick. of steel into wood
240 T=TH*ES/EW(J)
250      FOR I=1 TO 5
260 DW=HW(I)
270 LPRINT "Nom. size   38 x ";DW
280 SXW = 38*DW^2/6
290 MIW = SXW * DW/2
300 REM Print heading,set spacing of joist and cal.trib.loads
310 LPRINT"Joist at","No reinf.","Enhancer U","Enhancer1","Enhancer2"
320      FOR N=300 TO 600 STEP 100
330 IF N=500 THEN 820 ELSE 340
340 DL = .61*N/1000
350 W  = LL* N/1000
360 Q  = DL + W
370 QFACT = 1.25*DL + 1.5*W
380 SX = SXW :MI=MIW
390 GOSUB 890
400 LNO = ALLSP
410 REM Misc. geometry for enhancer after choice of flank.,s
420 REM U joist when flankage=0
430      FOR K=0 TO 2
440 IF K=0 THEN 740 ELSE 450
450 S = K*10
460 F = SQR (S^2 +(H-R)^2 - R^2)
470 AC = (ATN((H-R)/S))*180/PI
480 AB = (ATN(F/R))*180/PI
```

```
490 AA = 180-AC-AB
500 YY = SIN(AA*PI/180)*R
510 LF = (H - R - YY)/2
520 C=38+ 2*S
530 TL =2*(HH+F) + C
540 REM Calculate position of N.A. from the bottom of enhancer
550 REM then calc Mom. of Inertia of Enhancer to it's N.A.
560 Y=(2*HH*(H+HH/2)+2*F*(H-LF))/TL
570 Z1=(HH^3/12 + HH*(H+HH/2-Y)^2)*2
580 Z2=((H-R-YY)^3/12 + F*(H-LF-Y)^2)*2
590 Z3=C*Y^2
600 MIEN = (Z1 + Z2 + Z3)*T
610 REM Wood and transf. Steel Enh. together
620 D=DW+H  :AW=38*DW
630 X=(AW*DW/2+TL*T*(D-Y))/(AW+TL*T)
640 MI=MIW+AW*(X-DW/2)^2 +MIEN +TL*T*(D-X-Y)^2
650 SXC=MI/X
660 SXT=MI/(D-X)
670 IF SXC <SXT THEN 680 ELSE 690
680 SX=SXC  :GOTO 700
690 SX=SXT
700 GOSUB 890
710 LRE (K)=ALLSP
720 IF K=2 THEN GOTO 810
730     NEXT K
740 TL=2*HHHH+38-2*R
750 Y=HHHH*(HHH-HHHH/2)*2/TL
760 A1= (HH^3/12 + HH*(H-HH/2-Y)^2)*2
770 A2= (38-R)*Y^2 +(38-R)/12
775 A3=((H-R)^3/12 +(H-R)*(H/2-Y)^2)*2
780 MIEN = (A1+A2+A3)*T
790 GOTO 620
800 REM Print the results
810 LPRINT N,LNO,LRE(0),LRE(1),LRE(2)
820     NEXT N
840 RESTORE
850     NEXT I
860 NEXT J
870 STOP
880 REM Subroutine calculating max. allow. span
890 LDEF =(.59752*(EW(J)*MI/W)^(1/3))/1000
900 MRW = FBW(J)*KH*SX/10^6
910 MRL = .7*FB(J)*KZB(I)*KH*SX/10^6
920 WLW = SQR(8*MRW/Q)
930 LRL = SQR(8*MRL/QFACT)
940 IF LDEF < WLW THEN 950 ELSE 960
950 IF LDEF < LRL THEN 970 ELSE 980
960 IF WLW  < LRL THEN 990 ELSE 980
970 ALLSP=LDEF  :GOTO 1000
980 ALLSP=LRL   :GOTO 1000
990 ALLSP=WLW
1000 RETURN
1010 DATA SELECT,18.9,8.9,9300
```

```
1020 DATA "No.1",13.7,7.7,9300
1030 DATA "No.2",13.7,6.2,8400
1040 DATA "No.3",10.1,3.6,7500
1050 DATA 1.5,1.3,1.2,1.1,1
1060 DATA 89,140,184,235,286
1070 END
```

EFFECTS OF VARIATION OF JOIST SIZES, METAL THICKNESS AND DISTANCER

Table 1. Wood member 38x89 (2"x4")

S=12.5 Gauge	[mm] N.A.	[mm^4] [x10^6] Ix	[mm^3] [x10^5] Sxt	[mm^3] [x10^5] Sxc	[mm] Equiv.
16	65.00	5.12	1.40	7.89	117.00
18	62.70	4.98	1.28	7.95	116.00
20	59.70	4.75	1.14	7.95	114.00
22	57.80	4.55	1.04	7.88	113.00
S=25	N.A.	Ix	Sxt	Sxc	Equiv.
16	71.00	6.46	1.50	9.10	127.00
18	68.30	6.31	1.38	9.24	126.00
20	64.70	6.04	1.22	9.33	124.00
22	62.30	5.79	1.12	9.30	122.00
S=37.5	N.A.	Ix	Sxt	Sxc	Equiv.
16	77.10	8.08	1.63	1.05	137.00
18	74.00	7.90	1.50	1.07	136.00
20	69.80	7.58	1.34	1.09	134.00
22	67.00	7.29	1.23	1.09	132.00
S=50	N.A.	Ix	Sxt	Sxc	Equiv.
16	83.20	10.00	1.80	1.21	147.00
18	79.80	9.79	1.65	1.23	146.00
20	75.10	9.40	1.47	1.25	144.00
22	71.90	9.05	1.35	1.26	142.00
S=62.5	N.A.	Ix	Sxt	Sxc	Equiv.
16	89.30	12.40	1.99	1.38	157.00
18	85.60	12.00	1.82	1.40	156.00
20	80.50	11.50	1.62	1.43	154.00
22	77.00	11.10	1.49	1.44	152.00
S=75	N.A.	Ix	Sxt	Sxc	Equiv.
16	95.40	15.10	2.21	1.59	168.00
18	91.50	14.60	2.01	1.59	166.00
20	86.10	13.90	1.79	1.62	164.00
22	82.10	13.40	1.64	1.64	162.00

Table created by J. Janosik, P.Eng. Date:20.jan.1990
Note: Values in this Table are not factored!

Table 2. Wood member 38x140 (2"x6")

	[mm]	[mm^4] [x10^6]	[mm^3] [x10^5]	[mm^3] [x10^5]	[mm]
S=12.5 Gauge	N.A.	Ix	Sxt	Sxc	Equiv.
16	94.10	18.40	3.14	1.95	180.00
18	90.90	17.60	2.86	1.94	177.00
20	87.00	16.50	2.52	1.89	173.00
22	84.40	15.70	2.31	1.85	170.00
S=25	N.A.	Ix	Sxt	Sxc	Equiv.
16	101.00	21.70	3.36	2.16	190.00
18	96.70	20.90	3.06	2.16	188.00
20	92.00	19.60	2.69	2.13	184.00
22	89.00	18.60	2.45	2.09	180.00
S=37.5	N.A.	Ix	Sxt	Sxc	Equiv,
16	107.00	25.20	3.57	2.36	200.00
18	103.00	24.50	3.27	2.38	198.00
20	97.10	23.10	2.87	2.38	194.00
22	93.60	21.90	2.61	2.34	191.00
S=50	N.A.	Ix	Sxt	Sxc	Equiv.
16	113.00	29.10	3.79	2.57	209.00
18	109.00	28.80	3.48	2.61	208.00
20	102.00	26.90	3.07	2.63	204.00
22	98.40	25.60	2.79	2.60	201.00
S=62.5	N.A.	Ix	Sxt	Sxc	Equiv.
16	120.00	33.30	4.02	2.78	219.00
18	114.00	32.60	3.70	2.85	217.00
20	108.00	31.10	3.28	2.88	214.00
22	103.00	29.90	2.99	2.88	211.00
S=75	N.A.	Ix	Sxt	Sxc	Equiv.
16	126.00	37.90	4.26	3.01	229.00
18	120.00	37.20	3.93	3.09	227.00
20	113.00	35.60	3.50	3.15	224.00
22	108.00	34.10	3.20	3.15	221.00

Table created by J.Janosik, P.Eng. Date:20.jan.1990
Note:Values in this Table are not factored!

Table 3. Wood member 38x191 (2"x8")

	[mm]	[mm^4] [x10^6]	[mm^3] [x10^5]	[mm^3] [x10^5]	[mm]
S=12.5 Gauge	N.A.	Ix	Sxt	Sxc	Equiv.
16	122.00	43.70	5.34	3.58	240.00
18	118.00	41.50	4.85	3.51	236.00
20	114.00	38.50	4.28	3.39	230.00
22	111.00	36.50	3.94	3.29	226.00
S=25	N.A.	Ix	Sxt	Sxc	Equiv.
16	129.00	50.40	5.76	3.92	252.00
18	124.00	48.00	5.22	3.87	247.00
20	119.00	44.50	4.57	3.75	241.00
22	115.00	42.00	4.17	3.65	237.00
S=37.5	N.A.	Ix	Sxt	Sxc	Equiv,
16	135.00	57.20	6.14	4.23	262.00
18	130.00	54.80	5.56	4.21	259.00
20	124.00	50.90	4.86	4.11	252.00
22	120.00	48.00	4.42	4.01	248.00
S=50	N.A.	Ix	Sxt	Sxc	Equiv.
16	142.00	6.43	6.49	4.53	273.00
18	136.00	6.19	5.90	4.54	269.00
20	129.00	5.77	5.15	4.48	263.00
22	124.00	5.45	4.68	4.38	258.00
S=62.5	N.A.	Ix	Sxt	Sxc	Equiv.
16	149.00	71.60	6.82	4.82	283.00
18	142.00	69.30	6.22	4.87	280.00
20	134.00	65.00	5.45	4.84	274.00
22	129.00	61.50	4.95	4.76	269.00
S=75	N.A.	Ix	Sxt	Sxc	Equiv.
16	155.00	79.30	7.15	5.12	293.00
18	148.00	77.10	6.55	5.20	290.00
20	140.00	72.70	5.76	5.21	284.00
22	134.00	68.90	5.22	5.14	279.00

Table created by J.Janosik, P.Eng. Date:20.jan.1990
Note:Values in this Table are not factored!

Table 4. Wood member 38x242 (2"x10")

	[mm]	[mm^4] [x10^6]	[mm^3] [x10^5]	[mm^3] [x10^5]	[mm]
S=12.5 Gauge	N.A.	Ix	Sxt	Sxc	Equiv.
16	149.00	83.90	7.93	5.64	298.00
18	145.00	79.30	7.22	5.49	293.00
20	140.00	73.50	6.40	5.26	285.00
22	137.00	69.70	5.92	5.09	280.00
S=25	N.A.	Ix	Sxt	Sxc	Equiv.
16	156.00	95.60	8.60	6.13	311.00
18	151.00	90.40	7.77	6.00	306.00
20	145.00	83.40	6.82	5.76	297.00
22	141.00	78.60	6.25	5.57	292.00
S=37.5	N.A.	Ix	Sxt	Sxc	Equiv,
16	163.00	107.00	9.19	6.59	324.00
18	157.00	102.00	8.29	6.48	318.00
20	150.00	93.70	7.24	6.25	309.00
22	146.00	88.20	6.59	6.05	303.00
S=50	N.A.	Ix	Sxt	Sxc	Equiv.
16	170.00	119.00	9.72	7.01	335.00
18	163.00	113.00	8.79	6.94	329.00
20	155.00	105.00	7.65	6.74	321.00
22	150.00	98.30	6.94	6.54	314.00
S=62.5	N.A.	Ix	Sxt	Sxc	Equiv.
16	177.00	131.00	10.20	7.41	346.00
18	169.00	125.00	9.26	7.39	341.00
20	161.00	116.00	8.06	7.23	332.00
22	155.00	109.00	7.30	7.04	325.00
S=75	N.A.	Ix	Sxt	Sxc	Equiv.
16	183.00	143.00	10.70	7.80	356.00
18	176.00	137.00	9.71	7.83	351.00
20	166.00	128.00	8.47	7.71	343.00
22	160.00	120.00	7.66	7.54	336.00

Table created by J.Janosik, P.Eng. Date:20.jan.1990
Note:Values in this Table are not factored!

Max. Allowable Spans

Wood-commercial designation S-P-F

For uniform live load= 1.9 kN/m^2

GRADE = SELECT

Nom. size	38 x 89			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	1.980672	3.283482	3.547524	3.666309
400	1.79956	2.983246	3.223142	3.331061
600	1.57206	2.606104	2.815672	2.90995
Nom. size	38 x 140			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	3.115665	4.503132	4.738552	4.873298
400	2.830768	4.091366	4.305258	4.427679
600	2.472904	3.574139	3.76099	3.867937
Nom. size	38 x 184			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	4.094874	5.549796	5.778414	5.925317
400	3.72044	5.042324	5.250035	5.383511
600	3.250104	4.40488	4.586331	4.702925
Nom. size	38 x 235			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	5.229865	6.750029	6.978307	7.137615
400	4.751648	6.132802	6.340213	6.484955
600	4.150944	5.3575	5.538684	5.66513
Nom. size	38 x 286			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	6.36485	7.938133	8.169233	8.339324
400	5.782858	7.212268	7.422241	7.576779
600	5.05179	6.300497	6.48393	6.618923

GRADE = No.1

Nom. size	38 x 89			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	1.980672	3.283482	3.547524	3.666309
400	1.79956	2.983246	3.223142	3.331061
600	1.536148	2.504902	2.73465	2.815846
Nom. size	38 x 140			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	3.115665	4.503132	4.738552	4.873298
400	2.830768	4.091366	4.305258	4.427679
600	2.416412	3.430863	3.620793	3.712323
Nom. size	38 x 184			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	4.094874	5.549796	5.778414	5.925317
400	3.72044	5.042324	5.250035	5.383511
600	3.175856	4.238457	4.416005	4.515861
Nom. size	38 x 235			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	5.229865	6.750029	6.978307	7.137615
400	4.751648	6.132802	6.340213	6.484955
600	3.957346	5.043618	5.212283	5.318469
Nom. size	38 x 286			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	6.36485	7.938133	8.169233	8.339324
400	5.624081	6.942036	7.138747	7.272044
600	4.592043	5.668149	5.828762	5.937599

GRADE = No.2

Nom. size	38 x 89			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	1.9146	3.217546	3.4852	3.600522

400	1.68822	2.786477	3.047912	3.13733
600	1.378426	2.275149	2.488609	2.561619
Nom. size	38 x 140			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	3.011731	4.40258	4.649098	4.765656
400	2.655627	3.812746	4.026782	4.127179
600	2.168311	3.113094	3.287854	3.369828
Nom. size	38 x 184			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	3.958271	5.437976	5.66634	5.795038
400	3.490253	4.709426	4.908906	5.01865
600	2.849779	3.84523	4.008105	4.09771
Nom. size	38 x 235			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	5.053404	6.611729	6.839235	6.99434
400	4.45766	5.742803	5.937103	6.057276
600	3.639664	4.688979	4.847624	4.945745
Nom. size	38 x 286			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	6.15254	7.770715	8.001955	8.17041
400	5.425067	6.767082	6.961616	7.091508
600	4.429549	5.525299	5.684136	5.790192
GRADE = No.3				
Nom. size	38 x 89			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	1.485436	2.484649	2.724211	2.803187
400	1.286425	2.151769	2.359236	2.427631
600	1.050362	1.756912	1.926308	1.982152
Nom. size	38 x 140			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	2.33664	3.395456	3.589184	3.677261
400	2.02359	2.940551	3.108325	3.184602
600	1.652254	2.40095	2.537937	2.600217
Nom. size	38 x 184			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	3.071013	4.193457	4.373145	4.46949
400	2.659575	3.63164	3.787255	3.870692
600	2.171534	2.965222	3.092281	3.160406
Nom. size	38 x 235			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	3.922218	5.113099	5.287966	5.393897
400	3.39674	4.428074	4.579512	4.671252
600	2.773427	3.615507	3.739156	3.814061
Nom. size	38 x 286			
Joist at	No reinf.	Enhancer U	Enhancer1	Enhancer2
300	4.773422	6.023936	6.199342	6.314456
400	4.133906	5.216882	5.368788	5.46848
600	3.375319	4.259566	4.383597	4.464995

LIST OF REFERENCES

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TEST REPORT

ON

BENDING BEHAVIOUR OF DUO-JOISTS

submitted to

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by

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RGD-126

TEST REPORT ON BENDING BEHAVIOUR OF DUO-JOISTS

1. INTRODUCTION

This report is on the test results of Duo-Joists designed and assembled by the 800111 Ontario Inc. Duo-Joists are wood joists reinforced with a cold formed steel section, called an "enhancer," attached to the bottom of the wood joists. The test program was arranged through discussion with Mr. Jan Janosik, P.Eng.

2. TEST PROGRAM

2.1 Test Set-Up and Test Pressure

As shown in Figs. 1 and 2, the Duo-Joist was subjected a mid-span concentrated load in the Tinius-Olsen testing machine of the Applied Dynamics Laboratory of McMaster University. The Duo-Joist was simply supported at a clear span of 3.81 meter. The load from the test machine was applied to the Duo-Joist through a wooden loading shoe to prevent crushing of the wood fibres of the Duo-Joint, and to prevent possible lateral buckling of the Duo-Joist, especially those with initial twisting due to dry shrink of the wood.

Three LPDTs (Linear Potential Displacement Transducer) were used to monitor the vertical deflections of the Duo-Joist at the mid-span and the two ends. All the load and displacement outputs were fed into the computer controlled Optilog data acquisition system.

The load was applied continuously from zero to that corresponding to 25 mm deflection at the mid-span in about 3 minutes. In repeated load tests, the load was gradually increased until the mid-span deflection reached 25 mm and then it was gradually reduced to zero to complete one cycle of loading. For the repeated load tests, the above loading procedure were repeated 5 times.

2.2 Tests Performed and Observations

A total of 21 tests were carried out as listed in Table 1. Tests 3 and 15 were repeated tests subjected to 5 cycles of loading to 25 mm mid-span deflection. Tests 4, 7, 9, 12, 14 and 18 were retests of previously tested joists but after the enhancer had been removed.

Test 17 was performed to investigate the influence of prestressing on the behavior of the Duo-Joist. The Duo-Joist was first loaded with the enhancer at the top but not bonded to the wood joist. When the mid-span deflection reached to 26 mm at a load of 578 N, the enhancer was bonded to the wood joist as described in Table 2 for joist No. 12. The usual 38 mm overlap between the enhancer and the wood joist shown in Fig. 1 became 26 mm at one end, 54 mm at the mid-span and 18 mm at the other end of the enhancer. When the load was reduced to zero, the residual mid-span deflection was 20.1 mm due to the interaction between the enhancer and the wood joist.

The above prestressed Duo-Joist was turned to the normal position with the enhancer at the bottom and the above residual deflection now bowed upwards. When compressed down 26 mm at the mid-span, the load increased to 938 N compared to the 578 N prior to attaching the enhancer. Then load was reduced to zero with a residual deflection of 11.5 mm due to the buckling of the enhancer near the mid-span section. During an additional reloading stage, the load reached 1410 N at a mid-span deflection of 42 mm from the prebowed position.

Duo-Joists #4 and #11 were significantly twisted before the test, due to the uneven drying shrinkage. At the end of test, the two flanges of the enhancer of the Duo-Joist #2 buckling near the mid-span section as shown in Fig. 3. For Duo-Joist #9, the two flanges of the enhancer showed continuous local buckling over the central portion of the span and the horizontal slip between the end of the enhancer and the wood joist reached 1.6 mm when the mid-span deflection became 25 mm. It should be noted that the screws were 305 mm apart and that this type of buckling was observed for similarly bonded joists.

For Duo-Joist #6, the vertical slip between the enhancer and the wood joist reached 6.4 mm at the mid-span when the corresponding deflection became 25 mm. This large vertical

slip may be attributed to the fact that there were no screws or wood block at the mid-span between the enhancer and the wood joist.

Tests 19 and 21 were carried out on specimen with the ends of the U shaped enhancer bent up at a 30° angle over a length of 250 mm at each end to form a "truss type connection". In Test 19, the flanges (sides) of the enhancer were cut and bent out over a 50 mm length at mid span to represent conditions where bridging could interfere with attachment of the enhancer. In addition 38 mm square segments where cut in the sides of the enhancer vertically and bent out at 305 mm centre to centre spacing.

In Test 20 the enhancer was spliced with a 90 mm lap at 750, 1050 and 3050 mm from one end of the 3810 mm long enhancer. Five drywall screws were drifted through the laps to transfer tension forces between parts of the enhancer.

Test 21 used the same type of joist as Tests 17 and 18 (Joist 12) but without the 25 mm pre-bow. This allowed the wood block spacer to be used at mid span to create a kind of "King Post" configuration.

The ultimate load for Test 19 was controlled by tensile yielding of the enhancer at the location of the vertical cuts at mid span. Necking down of this section was visible after continued loading. Because the steel bearing plates at the supports were not used in this test, some crushing of the wood is included in the 25 mm mid-span deflection. Compared to Test No. 10, the load capacity at 25 mm was essentially the same indicating that the cut or notch in the enhancer did not significantly affect its stiffness.

Comparing Test 20 with Test 10 indicates that the splices were effective and no loss of stiffness occurred.

For Test 21, the improved end anchorage from the "truss" type of end connection of the enhancer plus the benefit of the wood spacer between the enhancer and the wood joint resulted in much improved stiffness compared to Tests 17 and 18.

2.3 Load-Deflection Results

The measured mid-span load-deflection curves are reproduced in Figs. 5 to 16. Some typical load-deflection data are also tabulated in Table 3.

The above test results indicate that the load-deflection behaviour of the Duo-Joists may be reasonably approximated as linear elastic when the mid-span deflection is less than 25 mm.

Duo-Joists with screws 76 mm apart are about 30% stronger than those with screws 305 mm apart, and the flanges of the former also suffer less severe local buckling.

The comparison of the load-deflection curves for those Duo-Joists with corresponding curves with the enhancer removed indicate that the enhancer roughly doubled the load for the wood joist at 25 mm mid-span deflection.

3. CONCLUSION

The following conclusions may be drawn from this experimental study:

1. The load carried by the wood joist at 25 mm deflection may be approximately doubled by the used of the enhancer.
2. Duo-Joists with screws 76 mm apart are about 30% stronger than those with screws 305 mm apart, and the flanges of the former also suffer less severe local buckling.
3. The load-deflection behaviour of the Duo-Joists may be approximated as linear elastic.
4. Prestressing is effective for reducing the net deflection, but the screws need to be more closely spaced to reduce the lateral buckling of the flanges of the enhancer.
5. No significant differences were observed between the nailed or screwed connections.
6. Load-deflection response is not significantly different for the Type ME and U enhancer, but the truss type connection seems to provide shifter response.

Table 1: Summary of Test Specimens and Type of Tests

Test No.	Joist No.	Wood$^{\Delta}$ Type	Wood Section (mm × mm)	Overlap of Enhancer and Wood (mm)	Enhancer Type	Load Type
1	1	#2	38 × 138	38	ME	Static
2	2	#2	37 × 138	38	ME	Static
3	3	#2	38 × 140	38	ME	5 Cycles
4	3	#2	38 × 140		Reference	Static
5	4	#1	38 × 140	38	ME	Static
6	5	#2	38 × 140	38	U	Static
7	5	#2	38 × 140		Reference	Static
8	6	#2	38 × 140	38	U	Static
9	6	#2	38 × 140		Reference	Static
10	7	#1	38 × 138	37	U	Static
11	8	#1	37 × 141	37	U	2 Cycles
12	8	#1	37 × 141		Reference	Static
13	9	#1	38 × 137	37	U	Static
14	9	#1	38 × 137		Reference	Static
15	10	#2	38 × 138	38	U	5 Cycles
16	11	#1	38 × 140	38	U	Static
17	12	#1	40 × 89	*	U	2 Cycles
18	12	#1	40 × 89		Reference	Static
19	6	#2	38 × 140	38	**	2 Cycles
20	9	#1	38 × 138	37	**	Static
21	12	#1	40 × 89	39	**	Static

NOTE:

Joist No. -- See Table 2

ME & U -- See Fig. 4

Reference -- With enhancer removed

* -- 26 mm at one end, 54 mm at middle, 19 mm at the other end

** -- Type U enhancer but bent up 30° from horizontal and 250 mm from the ends. For Test 19, the flanges of the enhancer were weakened by cuts of 50 × 89 mm at the mid-span, and cuts of 38 × 38 mm spaced 305 mm apart. For Test 20, the enhancer was spliced at 750, 1050, and 3050 mm from the left end of the enhancer with overlap length of 90 mm.

Δ #1 or #2 Wood Type refers to spruce species of #1 grade or #2 grade

Table 2: Details of Connection Between Wood Joist and Enhancer

BOND TYPE	JOIST NO.
Four nails at end, then the rest of nails at 305 mm o/c	1
Four nails at end, then the rest of nails at 305 mm o/c	2
Two screws at 76 mm o/c, and this pair at 305 mm apart	3
Two screws at 76 mm o/c, and this pair at 305 mm apart	4
Three screws at end; 2 screws and wood block at mid-span (King's Post)	5
Four screws at end; 2 screws and block at 600 mm from ends (Queen's Post)	6
Continuous 1 = screws at 76 mm o/c entire span	7
Continuous 2 = screws at 305 mm o/c	8
Continuous 2 = screws at 305 mm o/c	9
Continuous 3 = ring nails at 76 mm o/c	10
Gang nail at 305 mm o/c = 19 mm cross	11
Continuous 2 = screws at 305 mm o/c with 25 mm pre-bow	12

Table 3: Typical Mid-span Load-Deflection Data

Deflection (mm)	Load (kN)													
	*Test #1	Test #2	Test #3					Test #4	Test #5	Test #6	Test #7	Test #8	Test #9	Test #10
			Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5							
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0.841	0.664	1.159	0.982	1.142	1.125	1.014	0.254	1.025	0.722	0.361	0.960	0.543	0.984
10	1.470	1.329	2.014	1.805	1.980	2.025	1.786	0.684	1.901	1.361	0.821	1.933	0.943	1.831
15	2.118	2.024	2.871	2.695	2.812	2.804	2.672	1.178	2.694	2.056	1.336	2.908	1.385	2.736
20	2.795	2.723	3.866	3.566	3.730	3.719	3.560	1.616	3.426	2.647	1.818	3.827	1.886	3.584
25	3.478	3.422	4.976	4.671	4.903	4.852	4.664	2.090	4.250	3.231	2.310		2.192	4.445

* Refer to Table 1 for the description of the tests.

Table 3: Typical Mid-span Load-Deflection Data (Continued)

Deflection (mm)	Load (kN)															
	Test #11		Test #12	Test #13	Test #14	Test #15					Test #16	Test #17	Test #18	Test #19	Test #20	Test #21
	Cycle 1	Cycle 2				Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5						
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0.831	0.875	0.253	0.694	0.164	0.976	0.742	0.700	0.751	0.619	0.714	0.237	0.047	01.200	1.110	0.445
10	1.552	1.509	0.678	1.375	0.530	1.882	1.674	1.592	1.660	1.548	1.387	0.425	0.094	2.150	2.160	0.880
15	2.240	2.278	1.139	1.988	0.903	2.858	2.618	2.501	2.603	2.483	2.042	0.614	0.166	2.980	3.150	1.180
20	2.881	3.088	1.528	2.507	1.336	3.782	3.676	3.598	3.675	3.55	2.661	0.813	0.283	3.770	4.050	1.450
25	3.545	3.916	1.887	2.864	1.726	4.712	4.779	4.708	4.812	4.669	3.233	0.961	0.469	4.225	4.850	1.740

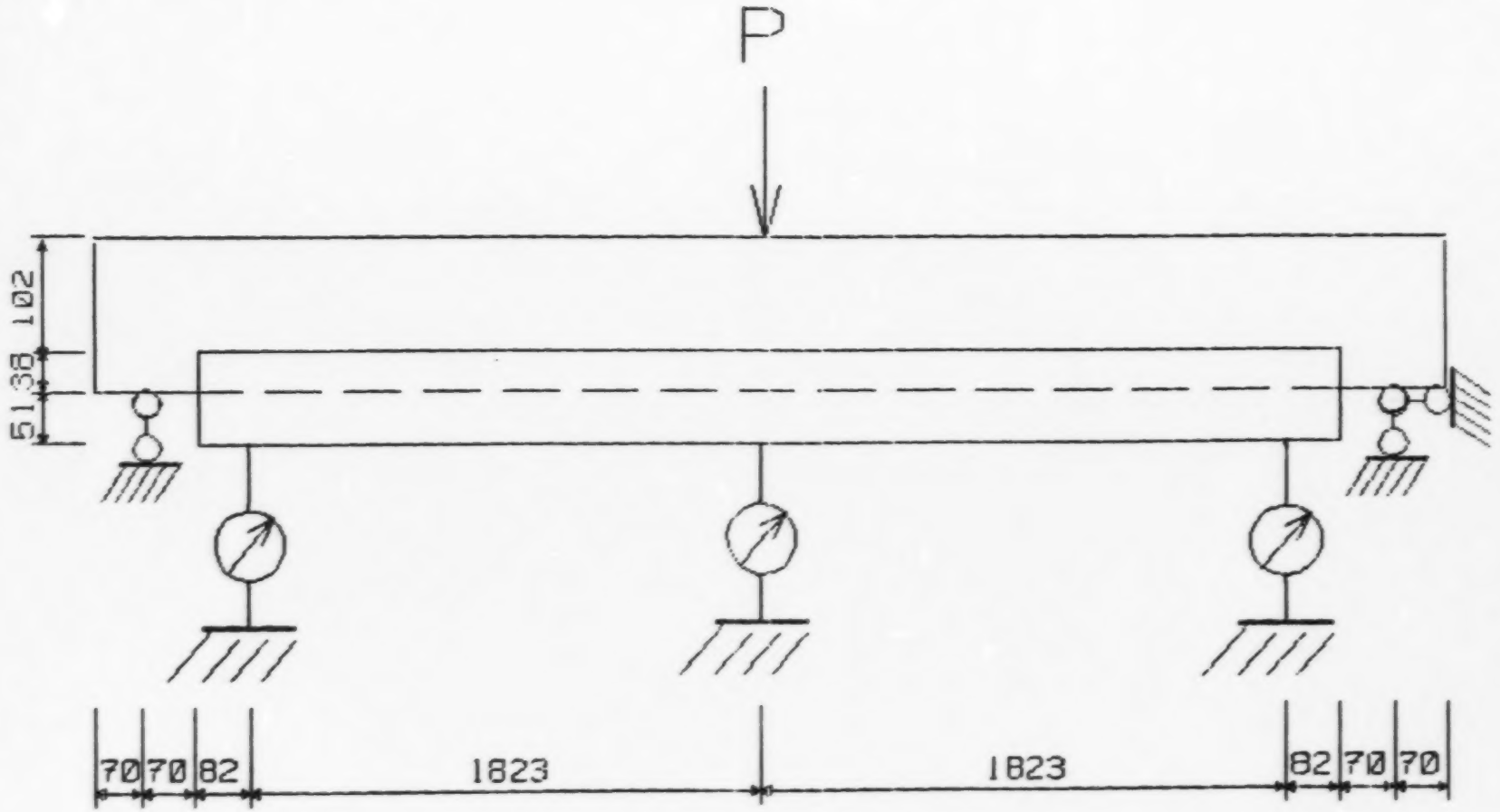

FIG. 1: TEST SET-UP FOR THE DUO-JOIST

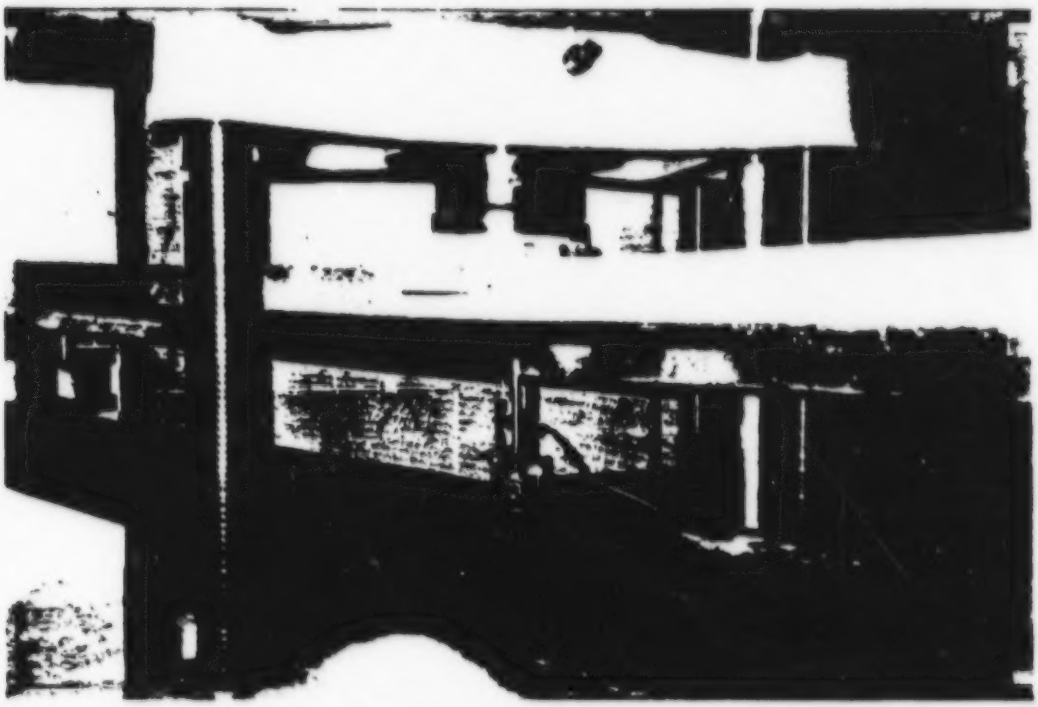

FIG. 2 TEST SET-UP FOR THE DUO-JOIST

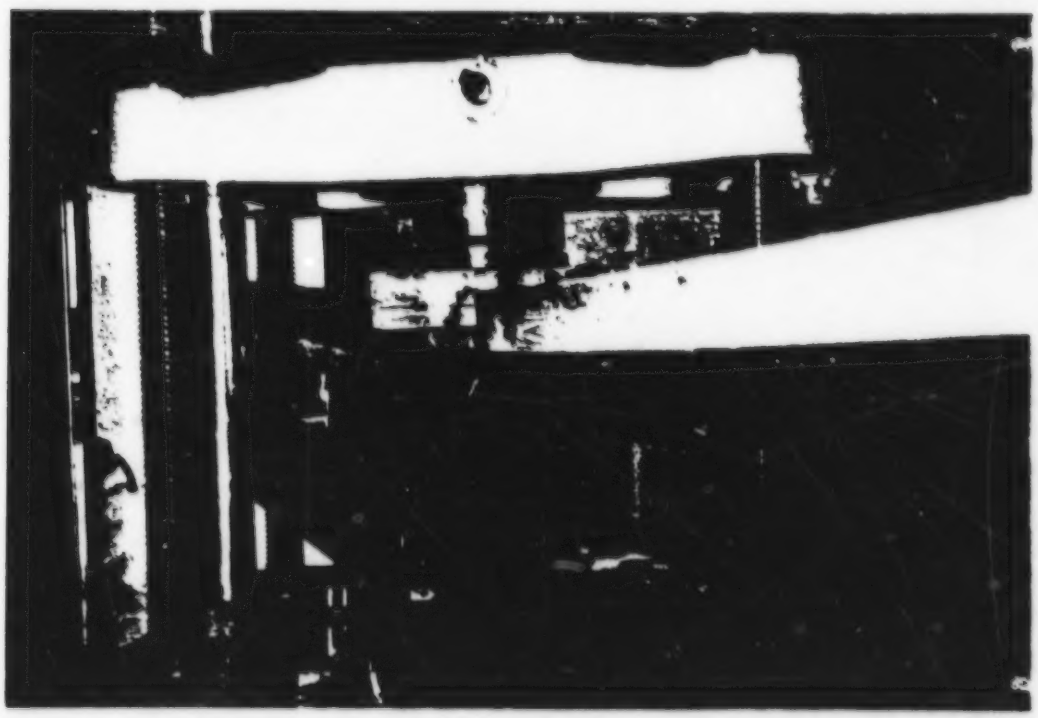

FIG. 3 LATERAL BUCKLING OF THE ENHANCER FLANGE

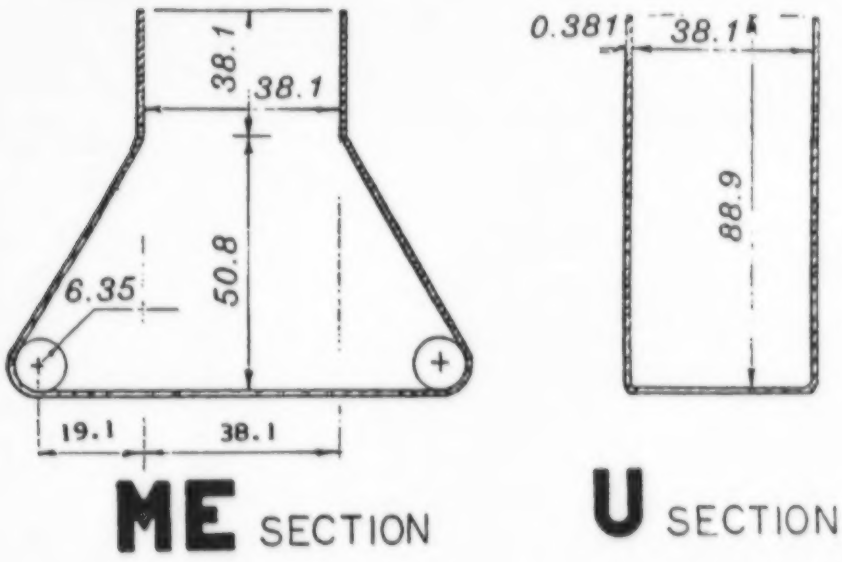

FIG. 4 CROSS SECTIONS OF THE ENHANCERS

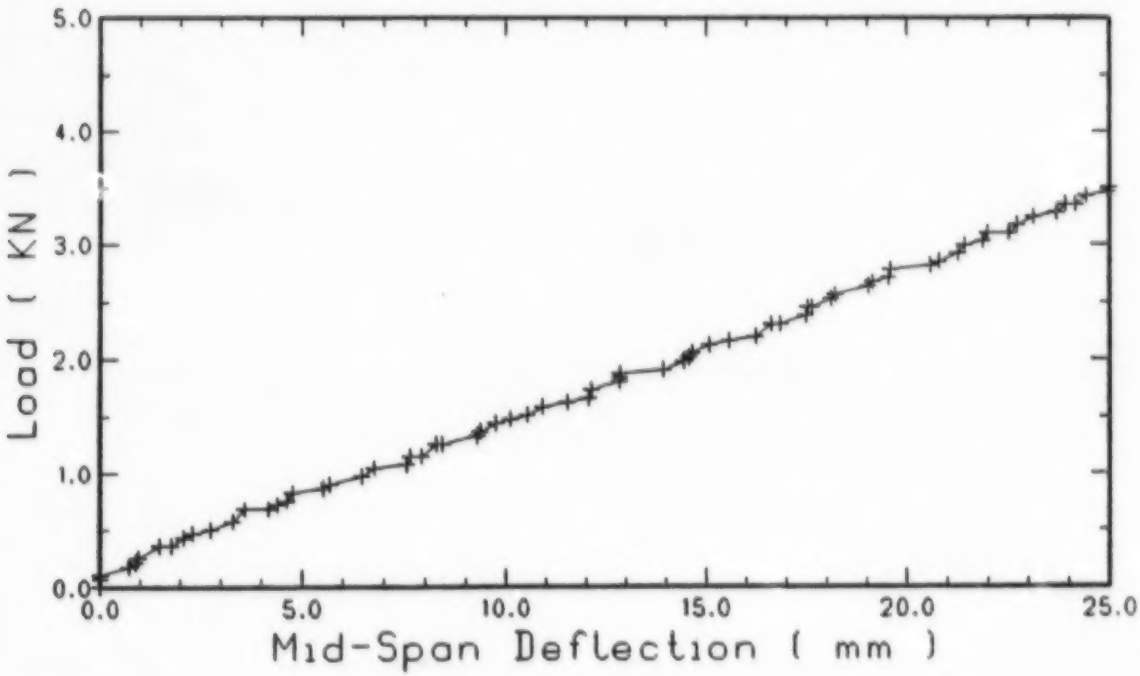

FIG. 5 LOAD-DEFLECTION DATA FOR DUO-JOIST #1

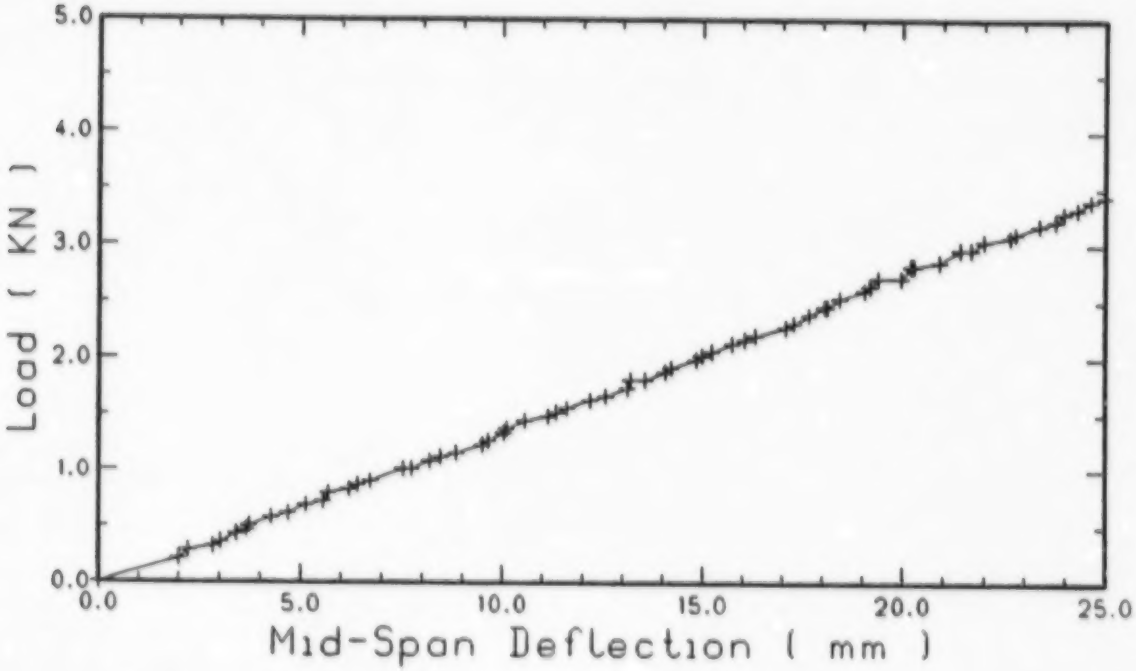

FIG. 6 LOAD–DEFLECTION DATA FOR DUO–JOIST #2

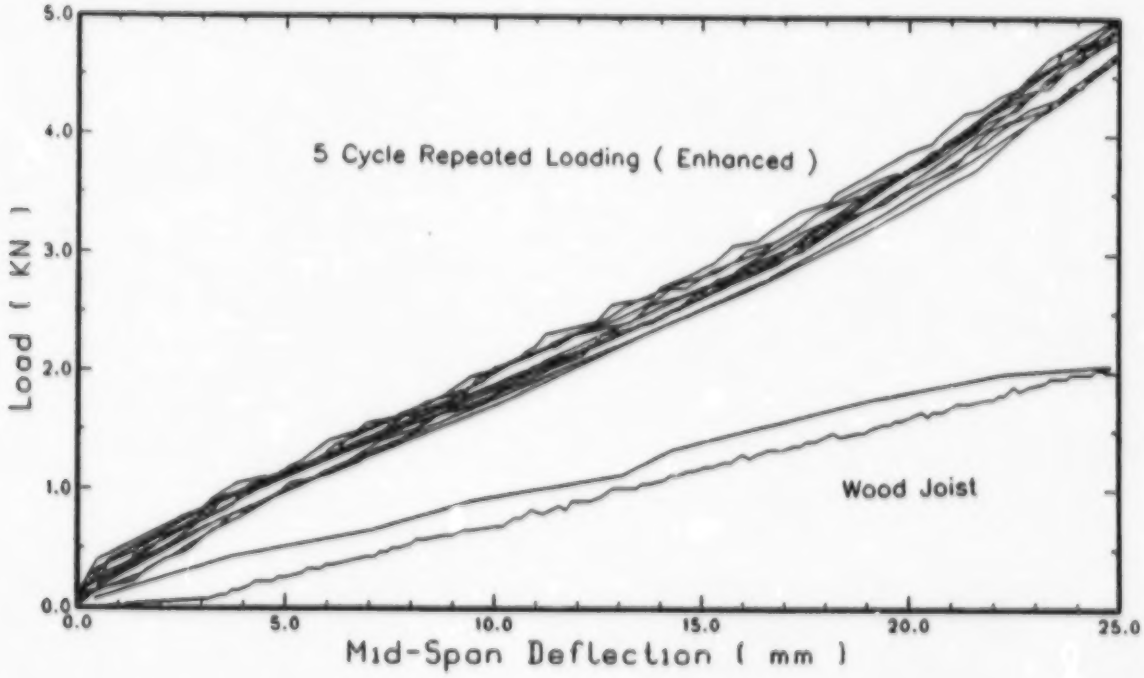

FIG. 7 LOAD–DEFLECTION DATA FOR DUO–JOIST #3

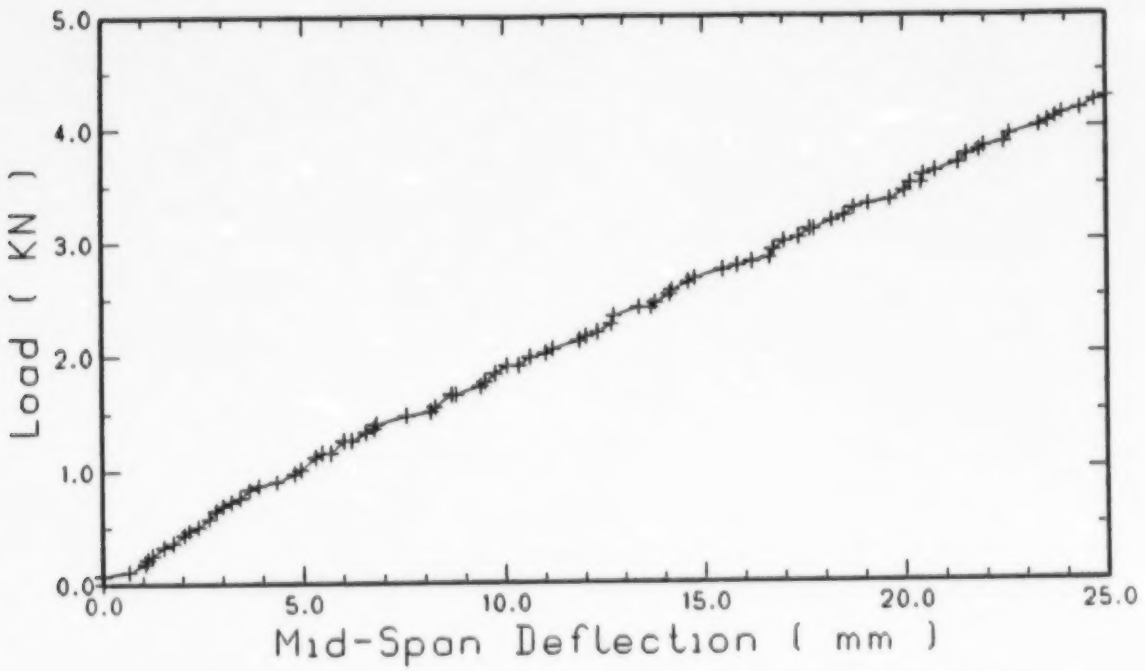

FIG. 8 LOAD–DEFLECTION DATA FOR DUO–JOIST #4

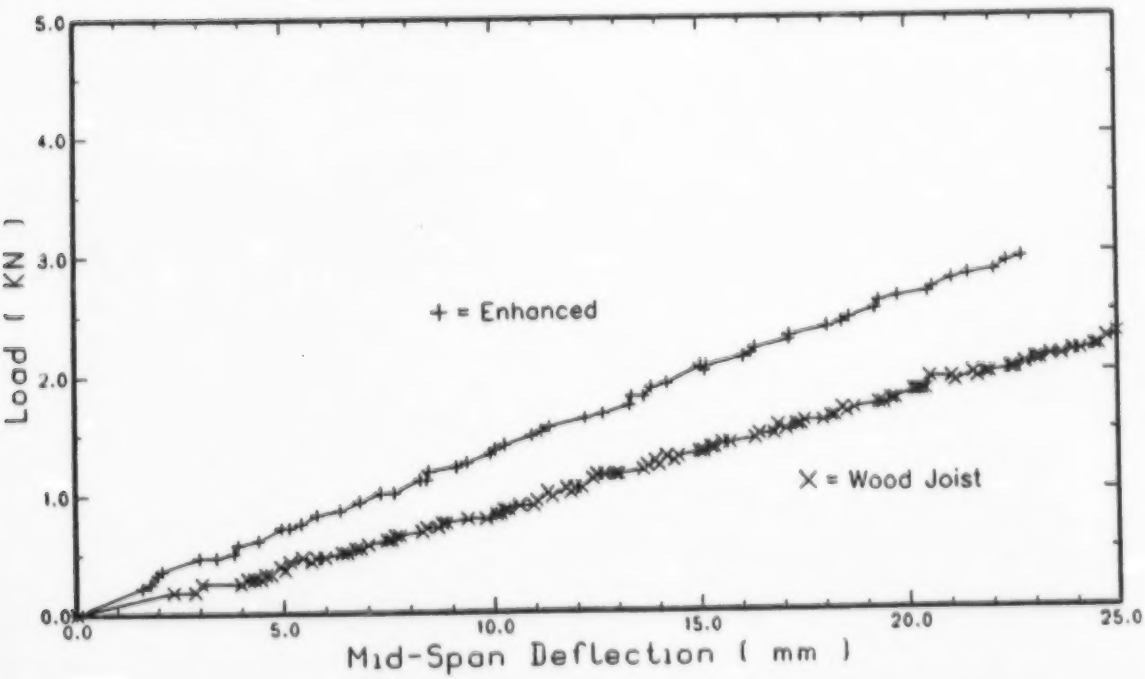

FIG. 9 LOAD–DEFLECTION DATA FOR DUO–JOIST #5

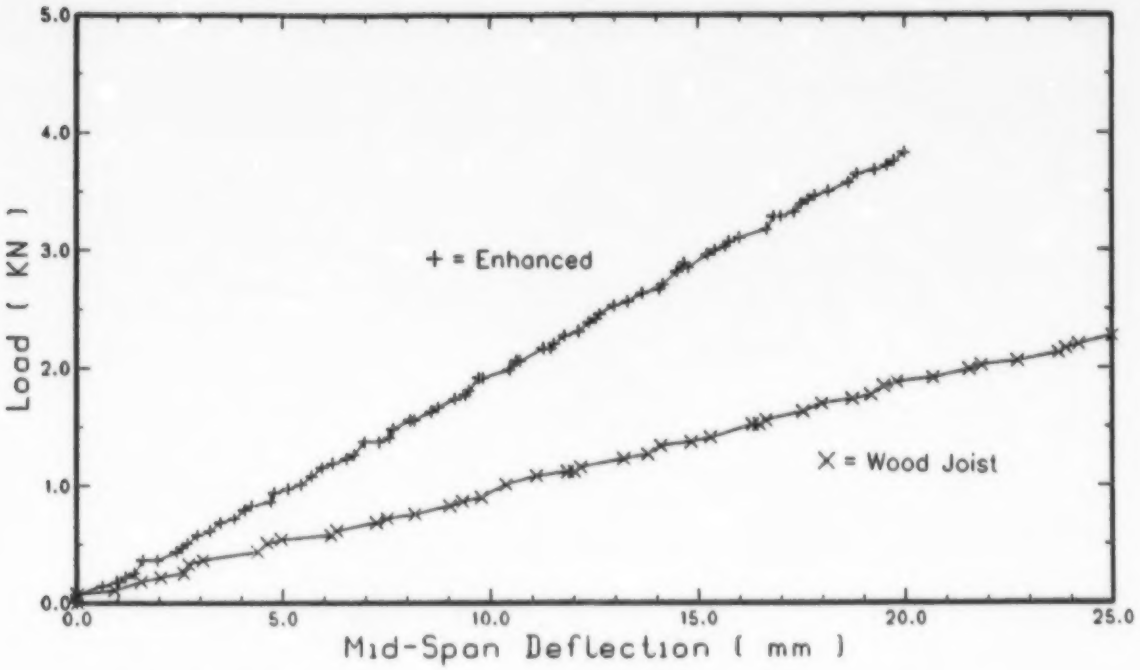

FIG. 10 LOAD–DEFLECTION DATA FOR DUO–JOIST #6

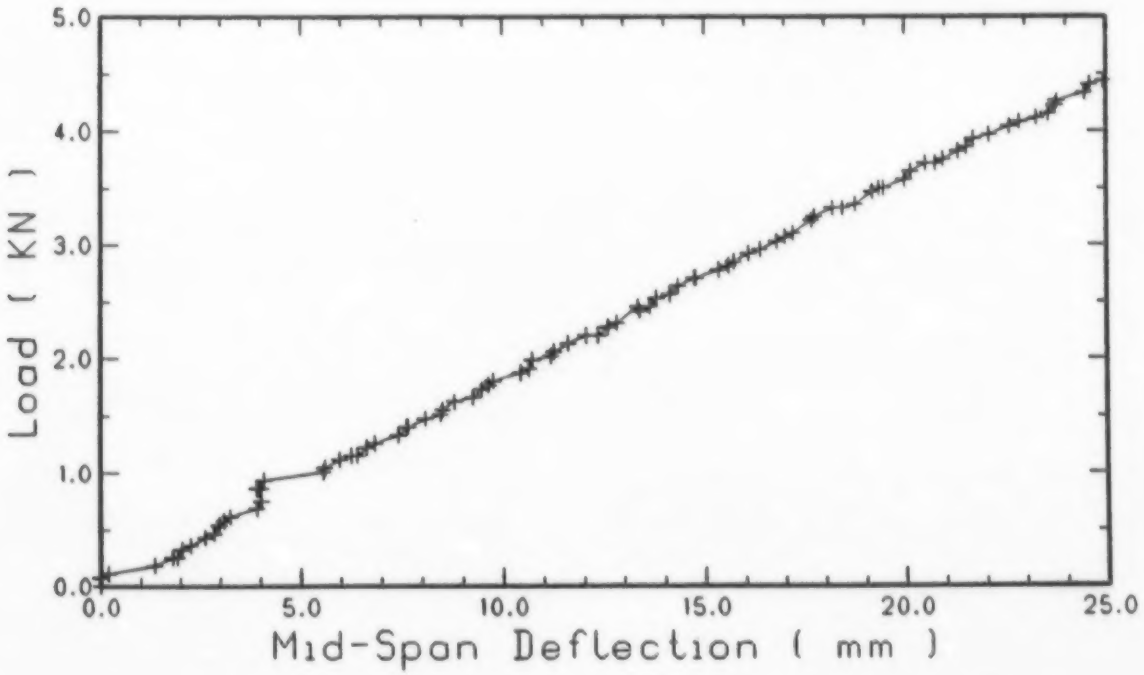

FIG. 11 LOAD–DEFLECTION DATA FOR DUO–JOIST #7

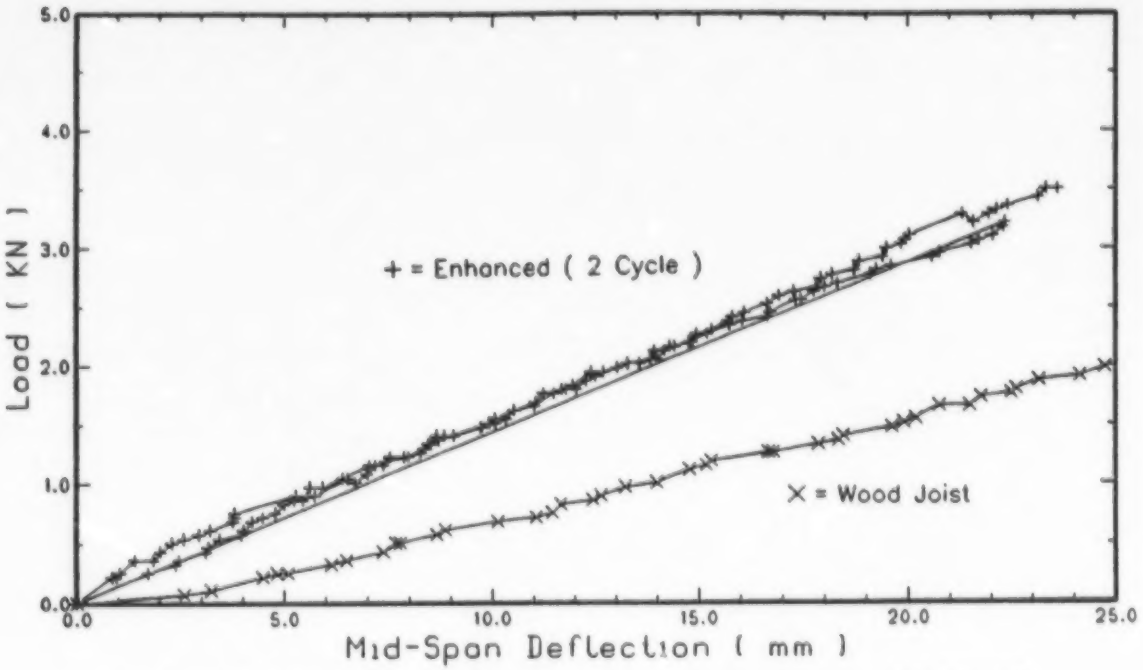

FIG. 12 LOAD–DEFLECTION DATA FOR DUO–JOIST #8

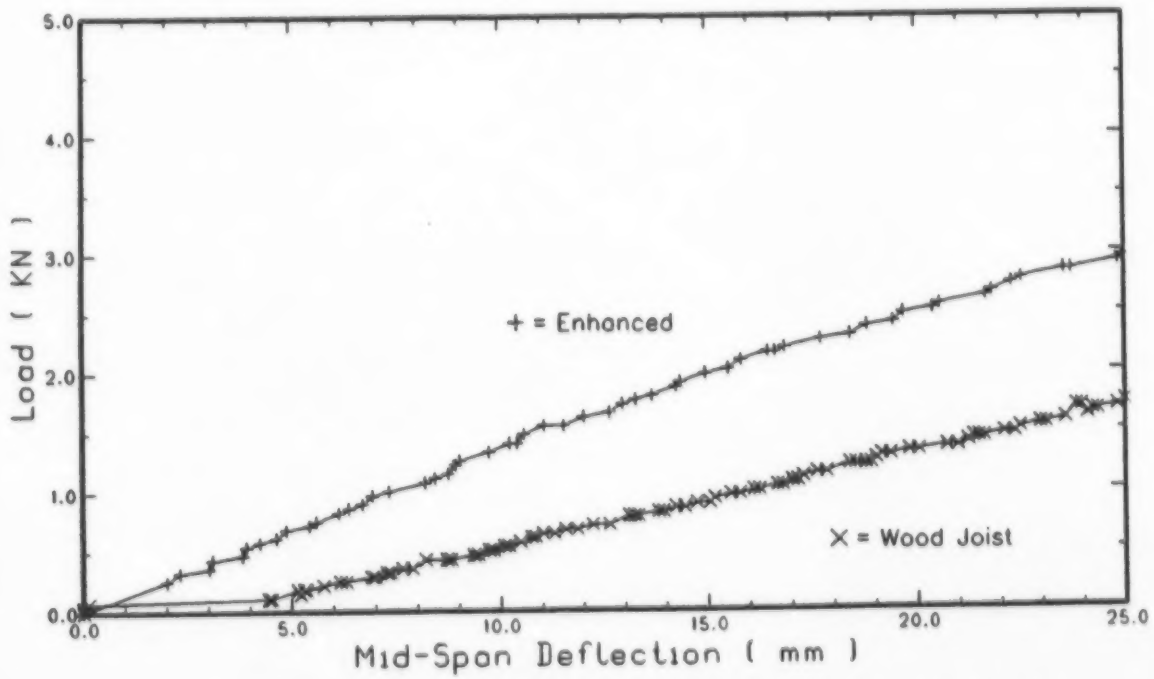

FIG. 13 LOAD–DEFLECTION DATA FOR DUO–JOIST #9

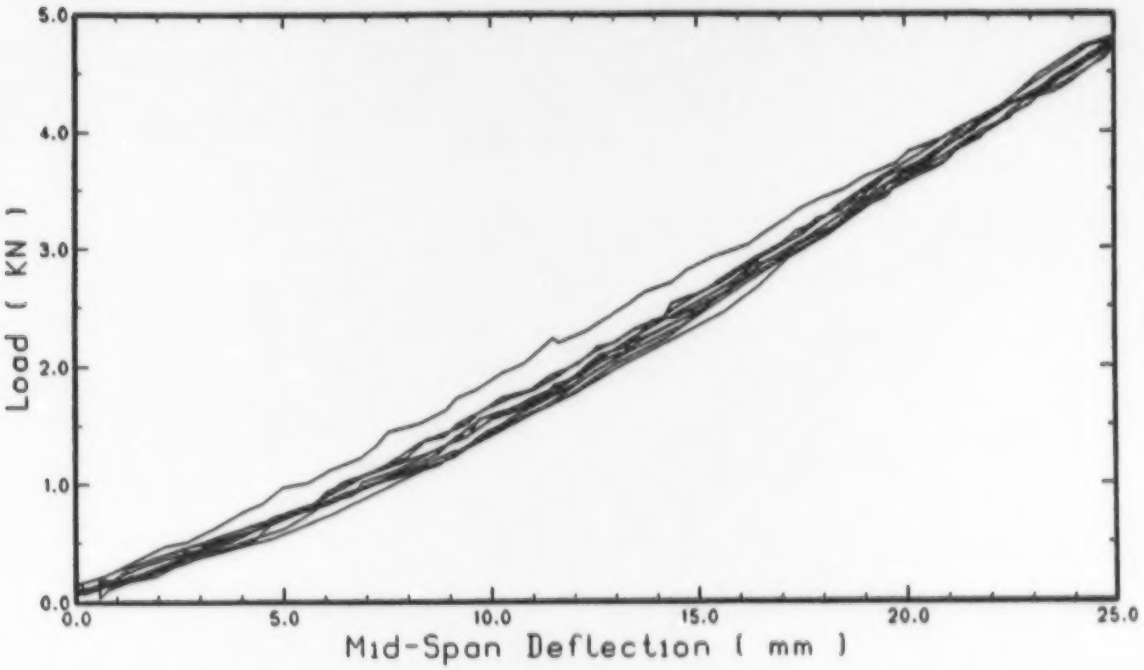

FIG. 14 LOAD–DEFLECTION DATA FOR DUO–JOIST #10

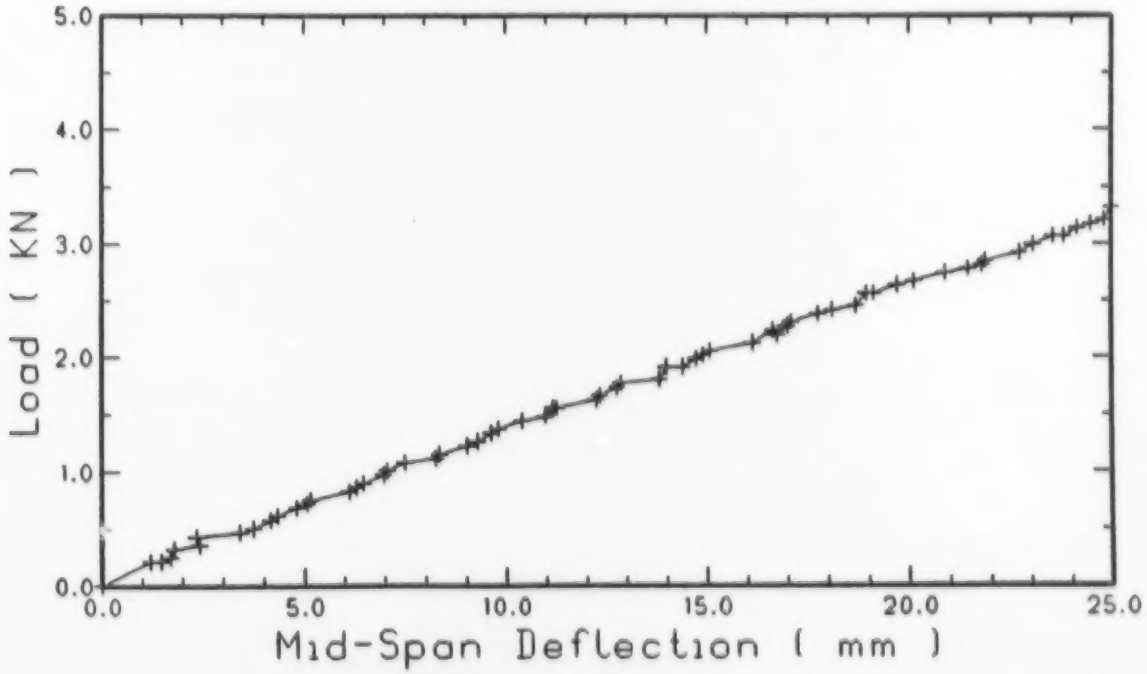

FIG. 15 LOAD–DEFLECTION DATA FOR DUO–JOIST #11

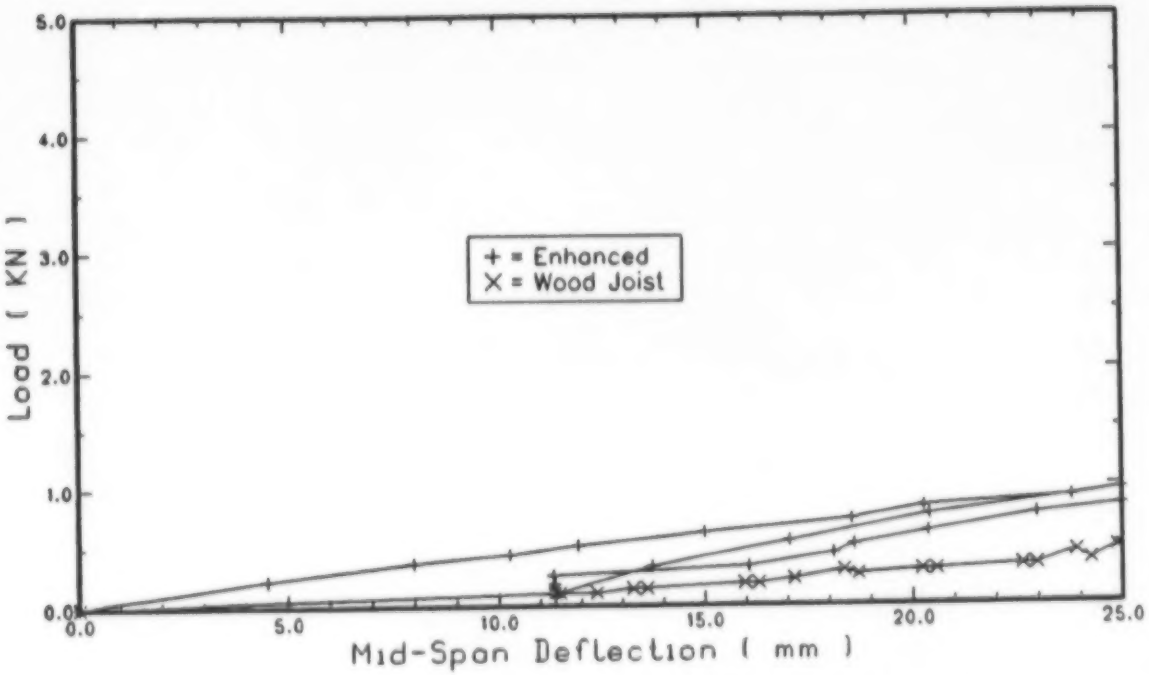

FIG. 16 LOAD–DEFLECTION DATA FOR DUO–JOIST #12

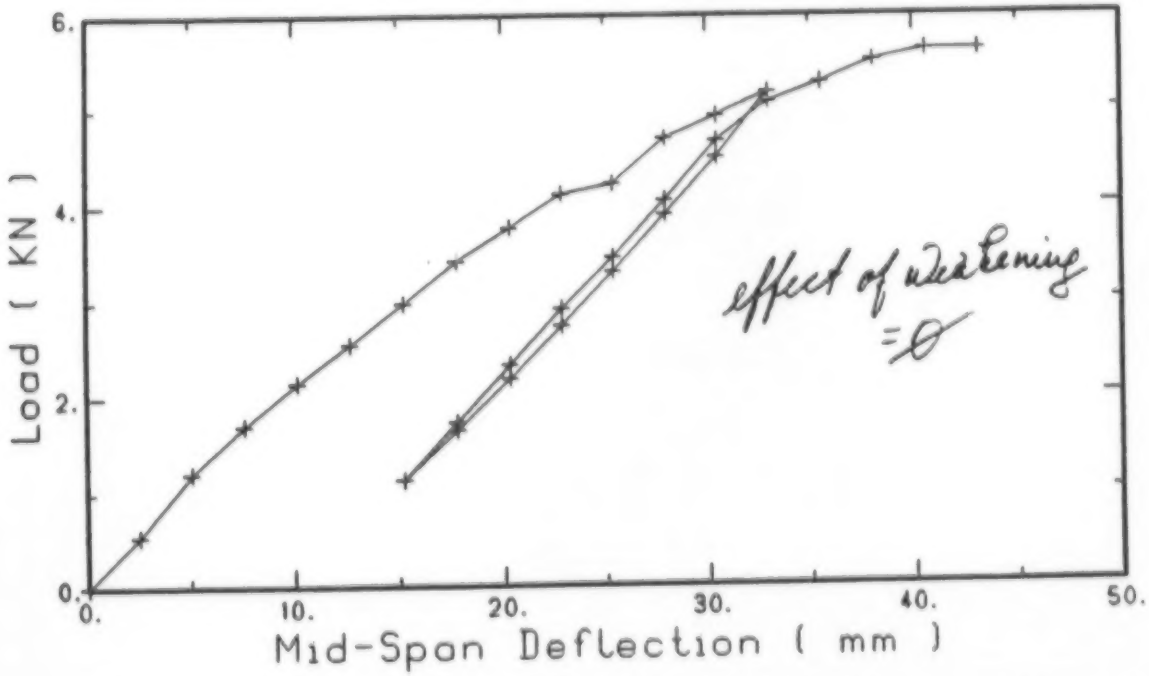

FIG. 17: LOAD–DEFLECTION DATA FOR TEST 19

See Joist #6 for comparison

Observation: Same load that causes 25 mm deflection in wood produces only 7.65mm deflection when the wood is equipped with bend ends enhancer

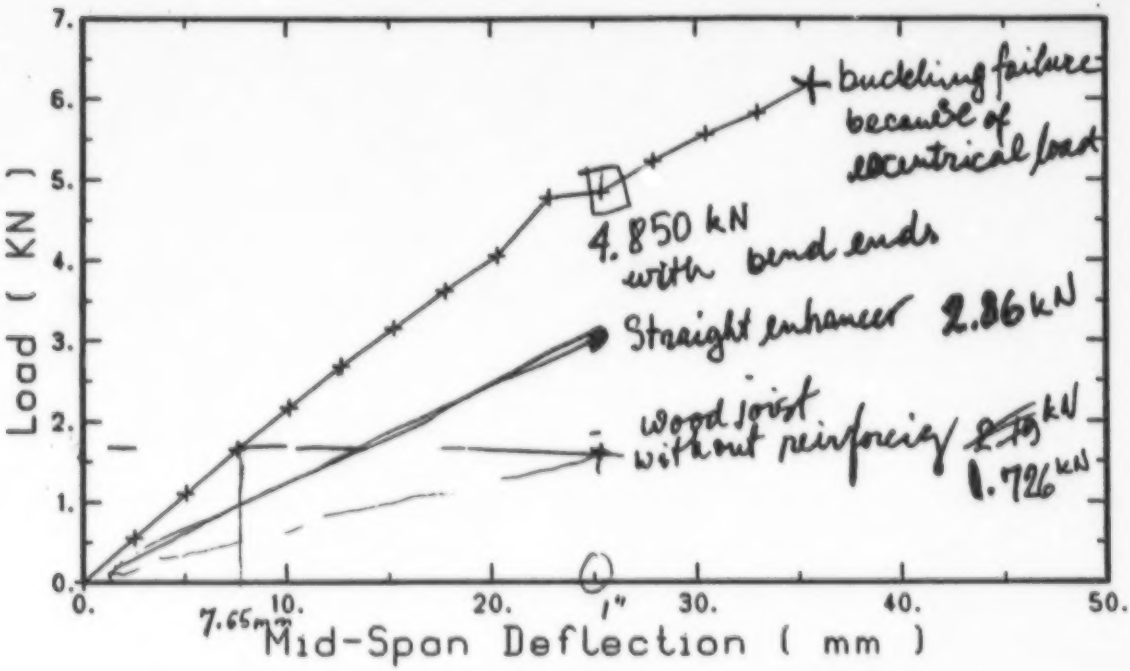

FIG. 18: LOAD–DEFLECTION DATA FOR TEST 20
see Joist # 9

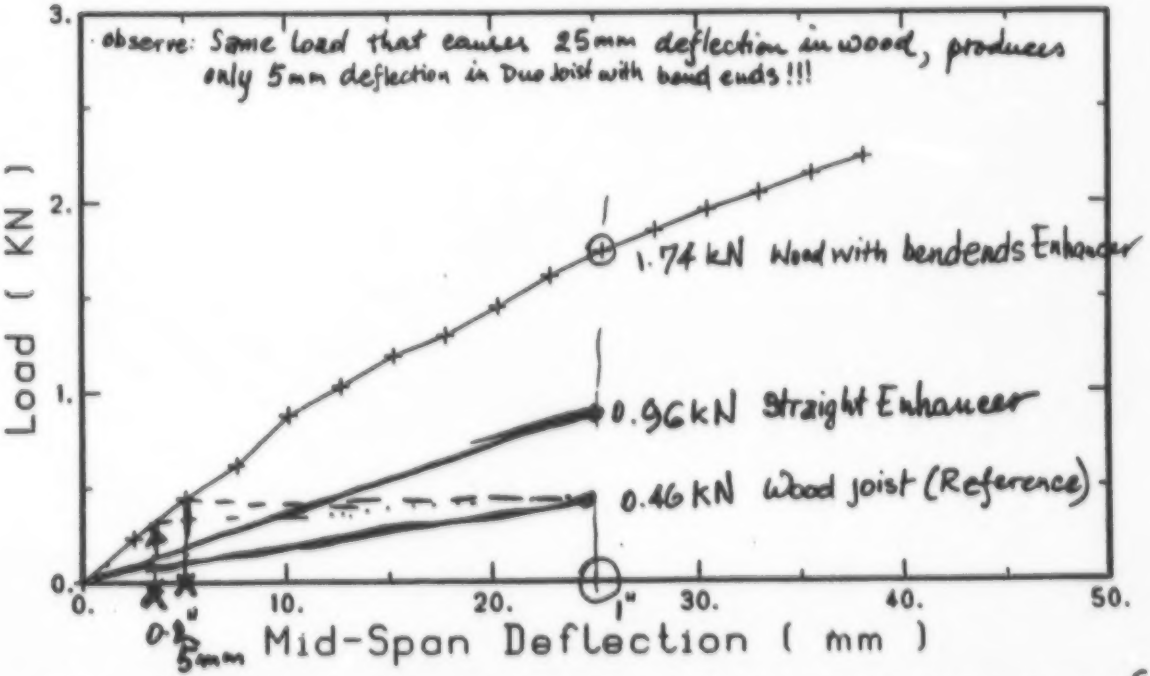
observe: Same load that causes 25mm deflection in wood, produces only 5mm deflection in Duo Joist with bend ends !!!
Load (KN)
Mid-Span Deflection (mm)
1.74 kN Wood with bendends Enhancer
0.96kN Straight Enhancer
0.46 kN Wood joist (Reference)
0.2" 5mm
1"

FIG. 19: LOAD–DEFLECTION DATA FOR TEST 21
Effect of different bond.
See joist # 12 for comparison